Superradiant Linear Raman Amplification in Plasma Using a Chirped Pump Pulse

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(Received 24 March 2005; published 14 October 2005)

A theoretical and numerical investigation of small-signal Raman backscattering from a chirped pump pulse in plasma shows that an ultrashort probe pulse will grow superradiantly, i.e., with an amplitude that scales with the propagation length while contracting self-similarly. These features are commonly associated with the nonlinear stages of Raman amplification in the pump depletion and Compton regimes. We show that the superradiant scaling results in very broad-bandwidth amplification due to gain distributed in frequency as well as spatially. Since different frequencies excite the plasma at different positions, wave breaking is avoided, and prepulses and pedestals are substantially suppressed. Linear chirped pulse amplification in plasma could provide a very broad-bandwidth alternative to solid state laser amplifiers, potentially usable for optical pulses a few cycles in duration.

DOI: 10.1103/PhysRevLett.95.165002 PACS numbers: 52.35.Mw, 42.50.Fx, 52.38.Bv, 52.59.Ye

High power short pulse lasers are valuable tools for scientists and industrialists exploring a wide range of phenomena and developing new technologies such as compact wakefield accelerators [1–3] and compact light sources [4]. The quest for reaching ever higher intensities is motivated by the possibility of creating particles from vacuum, which is predicted to occur above 10^{28} W/cm^2, for visible radiation. Modern high power lasers rely on chirped pulse amplification (CPA), a technique originally developed by Strickland and Mourou [5] in the 1980s to avoid damage to optical components. CPA usually involves using a monochromatic "pump" laser pulse to amplify a stretched frequency-chirped "probe" pulse in a suitable broadband amplifying medium, such as Ti:sapphire. After amplification the probe pulse is compressed to short duration and high power, currently up to several petawatts. Optical parametric chirped pulse amplification (OPCPA) [6] has been proposed as a way of increasing both the bandwidth and the power of laser amplifiers. However, both CPA and OPCPA amplifiers are limited by damage to optical components.

Amplification of laser pulses by stimulated Raman backscattering in plasma has been suggested as a way of achieving higher intensities without breakdown of the amplifying medium [7]. Raman backscattering in plasma is a parametric process where two transverse electromagnetic waves interact via longitudinal oscillations of the electron density, ponderomotively driven by the beat of the transverse waves. In homogeneous plasma, a low-amplitude probe can grow at the expense of a pump with frequency ω_0 if its frequency is down-shifted by the plasma frequency $\omega_p = \sqrt{n_0 e^2/\epsilon_0 m}$ from ω_0. Here, n_0 is the electron density, m and e their mass and charge, respectively, and ϵ_0 the permittivity of free space. Although the growth rate $\gamma_0 = a_0\sqrt{\omega_0\omega_p/2}$ (where $a_0 = eE_0/m\omega_0 c$ is the reduced vector potential of the pump, E_0 its electric field amplitude, and c the speed of light) can be high, the linear regime has generally been deemed unsuitable for amplification of short pulses due to the small frequency bandwidth $\sim\pi\gamma_0$ leading to lengthening of the probe pulse [8].

Also, the high gain may lead to unwanted amplification of radiation scattered off density fluctuations. As a possible way of avoiding this, the use of a longitudinal density profile with linearly varying plasma frequency, together with a frequency-chirped pump, has been investigated [9,10].

In order to obtain short amplified pulses, it has been suggested to make use of nonlinearities: First, for sufficiently high probe amplitude the pump will be depleted near the leading edge of the probe, so that growth is suppressed at its rear; due to Burnham-Chiao ringing [11], where energy is transferred both ways between pump and probe, the latter develops into a train of short pulses [7]. Second, when the ponderomotive potential becomes sufficiently strong for the oscillation frequency of the electrons in its valleys, $\omega_B = 2\omega_0\sqrt{a_0 a_1}$ (where a_1 is the reduced probe amplitude), to exceed the plasma frequency, the anharmonic character of the electron motion leads to an increased bandwidth for effective interaction. In this self-similar superradiant regime, the probe amplitude grows linearly with the propagation distance while contracting [12–14]. A similar increase of the amplification bandwidth due to transverse localization was found for the case of Raman scattering in deep plasma channels [15].

In this Letter, we investigate Raman amplification of a low-amplitude probe using a chirped pump. We find two important results: First, a large bandwidth pump allows short amplified pulses to display the characteristics of superradiant amplification: amplitude growth linear with the propagation distance, and self-similar contraction. And second, the amplitude of the longitudinal plasma wave remains limited, which avoids losses due to wave breaking. Although the analysis is performed for homogeneous plasma, it is similar to the case of linearly varying plasma frequency.

This superradiant scaling in the linear regime seems not to have been noticed in earlier investigations. Since every amplifier starting with a small seed has to start out in the linear regime [9], and all the experiments to date have used chirped pump pulses, it is, however, very relevant to their interpretation. Contrary to, e.g., Ref. [16], it is not sufficient to find superradiant growth to demonstrate having reached the nonlinear regime. Furthermore, the evolution in the linear regime determines the initial conditions for the transition into the nonlinear one.

We study the propagation of a low-amplitude transverse plane wave, the probe, along the positive z axis through plasma occupying the half-space $z \geq 0$, in the presence of a counterpropagating intense chirped pump wave. We describe the evolution of the transverse electromagnetic fields of pump and probe by the wave equation (denoting partial time and z derivatives with ∂_t and ∂_z, respectively)

$$(\partial_t^2 - c^2\partial_z^2 + \omega_p^2)\vec{a} = -\omega_p^2\vec{a}\,\delta n/n_0 \qquad (1)$$

for the normalized (in units of mc/e) vector potential $\vec{a}(z, t)$, and of the plasma wave by the oscillator equation

$$(\partial_t^2 + \omega_p^2)\delta n/n_0 = \partial_z^2\phi_p/m \qquad (2)$$

for the deviations $\delta n(z, t)$ of the electron density from its unperturbed homogeneous value n_0, driven by the ponderomotive potential $\phi_p \approx mc^2\vec{a}^2/2$.

Corresponding to the separate fast and slow time scales of transverse waves and plasma oscillations, respectively, we write the vector potential as $\vec{a}(z, t) = (a_0 e^{i\varphi_0 + i\varphi_{\rm ch}} + a_1 e^{i\varphi_1})\vec{e}_+/2 + \text{c.c.}$, where $\vec{e}_+$ is a unit vector for circular polarization, $a_0(z, t)$ the envelope of the pump, $a_1(z, t)$ that of the probe, i.e., seed and scattered wave; the respective phases are $\varphi_0(z, t) = \omega_0(t + z/c)$, $\varphi_{\rm ch} = \alpha(t + z/c)^2/2$, and $\varphi_1(z, t) = \omega_1(t - z/c)$. The pump frequency $\omega_0 + \alpha(t + z/c)$ is chirped at a rate α. The corresponding ponderomotive potential is $\phi_p(z, t) \approx (a_0^* a_1 e^{-i\delta\varphi - i\varphi_{\rm ch}} + \text{c.c.})mc^2/4$, with $\delta\varphi(z, t) = \varphi_0 - \varphi_1 = (\omega_0 - \omega_1)t + (\omega_0 + \omega_1)z/c$. Similarly, we define the envelope $n(z, t)$ for the density modulations $\delta n/n_0 = ne^{-i\delta\varphi}/2 + \text{c.c.}$ The definition of probe and density envelopes with respect to unchirped phases means that variations due to the chirp of the pump will be taken into account in these envelopes.

Neglecting dispersion effects due to the linear plasma response, and assuming a small chirp rate α, we find

$$(\partial_t - c\partial_z)a_0 = i\omega_p^2 n a_1 e^{-i\varphi_{\rm ch}}/4\omega_0, \qquad (3)$$

$$(\partial_t + c\partial_z)a_1 = i\omega_p^2 n^* a_0 e^{i\varphi_{\rm ch}}/4\omega_0, \qquad (4)$$

$$(\partial_t^2 + \omega_p^2)n = -2\omega_0^2 a_0 a_1^* e^{i\varphi_{\rm ch}}. \qquad (5)$$

The second order equation for the density is retained to allow for energy flow in either direction between the transverse waves (a possibility that does not occur for monochromatic pump in inhomogeneous plasma). These equations have been solved numerically below.

It is common to make progress in the analytic description by assuming that the frequencies are near the resonance where energy flows from pump to probe. Further, for small probe amplitude a_1 the pump amplitude a_0 remains constant, and the envelope equations for probe and density modulations simplify to

$$\partial_\zeta a_1 = \beta\gamma n, \qquad \partial_\tau n = \gamma^* a_1/\beta, \qquad (6)$$

where we have changed the variables to $\zeta = z/c$, $\tau = t - z/c$, and $\beta = (\omega_p/4\omega_0)^{3/2}$ is the probe amplitude at which, in the resonant case, the plasma wave breaks, $|n| = 1$ (for $\omega_p \approx 5 \times 10^{13}\ \text{s}^{-1}$, $\omega_0 \approx 2.4 \times 10^{15}\ \text{s}^{-1}$, $\beta \approx 3.8 \times 10^{-4}$). The coefficient $\gamma = \gamma_0 e^{i\varphi_{\rm ch}}$ contains both the growth rate $\gamma_0(\zeta, \tau) = a_0(\zeta, \tau)\sqrt{\omega_0\omega_p/2}$, and the chirped phase $\varphi_{\rm ch} = \alpha(2\zeta + \tau)^2/2$ of the pump.

A field $\vec{A}_S(t)$ applied at the plasma edge, $z = 0$, will propagate into the plasma and, if the plasma dispersion can be neglected, preserve its shape. Where it meets the pump, it will generate density modulations and thus act as a seed for scattering. We therefore split the probe envelope $a_1 = \beta[b(\zeta, \tau) + s(\tau)]$ into seed $s = eA_S(\tau)/\beta mc$ and scattered pulse b (scaled with the wave breaking limit β). Since pump and probe interact efficiently only near resonance, the chirp of the pump is imprinted on the density modulations and the scattered probe. Thus, absorbing the phase factor of γ in modified envelopes $\tilde{n} = ne^{2i\alpha\zeta(\zeta+\tau)}$, $\tilde{b} = be^{-i\alpha\tau^2/2}$, $\tilde{s} = se^{-i\alpha\tau^2/2}$ simplifies the coupling in Eqs. (6) to

$$\partial_\zeta\tilde{b} = \gamma_0\tilde{n}, \qquad (\partial_\tau - 2i\alpha\zeta)\tilde{n} = \gamma_0^*(\tilde{b} + \tilde{s}). \qquad (7)$$

Formally, we find Green's functions $\tilde{b} = g$ for the probe, and $\tilde{n} = h$ for the density by replacing the seed with a δ function, $\tilde{s}(\tau) = \delta(\tau)$. The response to arbitrary seeds may be expressed in the form of convolutions, but for short seeds it may be approximated by the Green's functions themselves. Eliminating the probe g yields $(\partial_\zeta\partial_\tau - 2i\alpha\zeta\partial_\zeta - 2i\alpha - |\gamma_0|^2)h = 0$, with boundary condition $h(\zeta, \tau = 0+) = \gamma_0^*$. Following the case of unchirped pump, we look for a self-similar solution depending only on the product $x = 2\alpha\zeta\tau$, and obtain, with $q = |\gamma_0^2|/2\alpha$,

$$[xd_x^2 + (1 - ix)d_x - i - q]h(x) = 0, \qquad h(0) = \gamma_0^*, \qquad (8)$$

the solution of which is a Laguerre function [17]:

$$h(x) = \gamma_0^* L_{iq-1}(ix). \qquad (9)$$

This Green's function has been found for the similar case of inhomogeneous plasma [10,18]. Using Eqs. (7) we find the probe Green's function:

$$g(\zeta, \tau) = |\gamma_0|^2\zeta G(q; x), \qquad (10)$$

with $G(q;x) = i[L_{iq}(ix) - L_{iq-1}(ix)]/x$. The z dependence of the Green's functions (amplitude) is shown in Fig. 1 for different times.

Although—taken on their own—they would violate the assumption of a slowly varying amplitude, the Green's functions show features that should still hold for finite duration seeds: As is apparent from Fig. 1, the density modulation amplitude is limited; far from both plasma edge and seed position, it tends to a finite value $|h| \to h_{\lim} = |\gamma_0|\sqrt{(e^{2\pi|q|}-1)/(2\pi|q|)}$ $(\approx \sqrt{\alpha/\pi}e^{\pi|q|}$ for $|q| > 1)$. The probe g, Eq. (10), has a maximum amplitude $g_{\max} = |\gamma_0|^2\zeta G_{\max}(q)$ close to the seed position, with $G_{\max}(q) = \max_x|G(q;x)|$ (Fig. 2). It grows linear with ζ and thus the interaction time t, while its width decreases $\propto 1/t$. These features are very similar to the cases of pump depletion for unchirped pump [7], and of superradiant growth [12].

These results can be understood by considering the distributed gain: consider a seed with Fourier components s_ω within a bandwidth Δ_s. Each frequency component s_ω excites density modulations resonantly where its beat with the chirped pump matches the plasma frequency. Since the approximately exponential growth of the modulations, with rate $\sim|\gamma_0|$, is restricted by the time $\sim\pi|\gamma_0/2\alpha|$ in which the seed passes through the Raman bandwidth $\sim\pi|\gamma_0|$ the modulation amplitude grows to $h_{\lim}|s_\omega|$. Assuming that the scattered Fourier components, of approximately constant magnitude, superpose coherently without phase difference at some short distance behind the seed and that the density modulation amplitude there is close to the limiting value, the first of Eqs. (7) corresponds to linear growth of the scattered amplitude, $|b_{\max}| = |\gamma_0 n|\zeta \approx |\gamma_0 s_\omega|h_{\lim}\zeta$. However, deviations from these assumptions reduce the growth to $|b_{\max}| = |\gamma_0^2 s_\omega|G_{\max}\zeta$. Further behind the seed, the scattered amplitude drops rapidly due to dephasing. The distance of linear growth is limited by the seed bandwidth, Δ_s, to $\zeta \le \Delta_s/2|\alpha|$, and thus the gain factor to $|b_{\max}/s| \le |q|G_{\max}$, where $s = \Delta_s|s_\omega|$ is an approximate measure of the seed amplitude.

Breaking of the plasma wave ($|n| = 1$) can be avoided by keeping $|s_\omega| < 1/h_{\lim}$. Since in the Raman interaction energy flows from high to low frequencies, the maximum useful seed bandwidth is ω_p, thus $s \le \omega_p/h_{\lim}$, and $|b_{\max}| \le |q|G_{\max}\omega_p/h_{\lim}$. For typical values $a_0 = 0.004$, $\omega_p = 0.02\omega_0$, thus $|\gamma_0| = 0.02\omega_p$; $|\alpha| = 10^{-4}\omega_p^2$, thus $q = 2$, the gain factor can reach $|q|G_{\max} \approx 0.45h_{\lim}/|\gamma_0| \approx 68$, and the probe amplitude can exceed the wave breaking limit for the resonant case, β, by a factor of $|q|G_{\max}\omega_p/h_{\lim} \approx 22$, corresponding to $a_1 = 0.008$. For $\omega_p = 0.06\omega_0$, leaving a_0 and α unchanged, we find $q = 6$, a gain factor of 1.5×10^7, and $a_1 \le 50\beta \approx 0.09$. Although this is beyond the linear regime treated in this Letter, it serves to show that the present considerations are relevant right up to the nonlinear regime.

In order to assess the limitations of the analytic results, we have solved Eqs. (3)–(5) numerically, using a Crank-Nicholson type algorithm. Figure 3 shows snapshots of the evolution of the probe and density modulations for a seed of finite duration. Whereas those of the scattered field look very similar to the Green's function, narrowing while growing in amplitude, the corresponding density modulations differ slightly. Apparently, detuning from the plasma resonance does not completely suppress further excitation of the plasma wave. Although efficiency is not of central importance for this Letter, we estimate it to be $\sim$8% for this case, by comparing the energy gained by the probe with that encountered in the pump, for a seed amplitude just avoiding wave breaking ($|n| \le 1$).

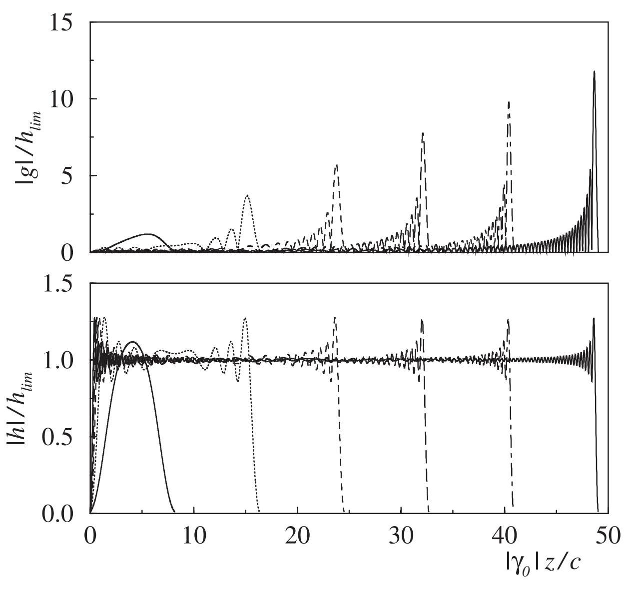

FIG. 1. Snapshots of Green's functions (amplitude) for probe vector potential, g (top), and density modulations, h (bottom), normalized with $h_{\lim}$; $q = 1.86$.

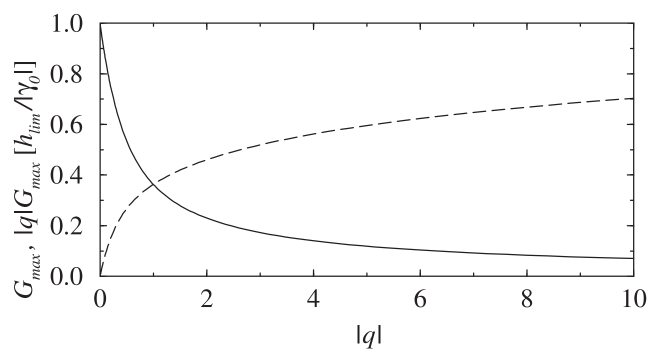

FIG. 2. Linear amplitude growth rate $G_{\max}$ (solid line) as a function of $|q| = |\gamma_0^2/2\alpha|$, normalized with $h_{\lim}/|\gamma_0|$; also $|q|G_{\max}$ (dashed line).

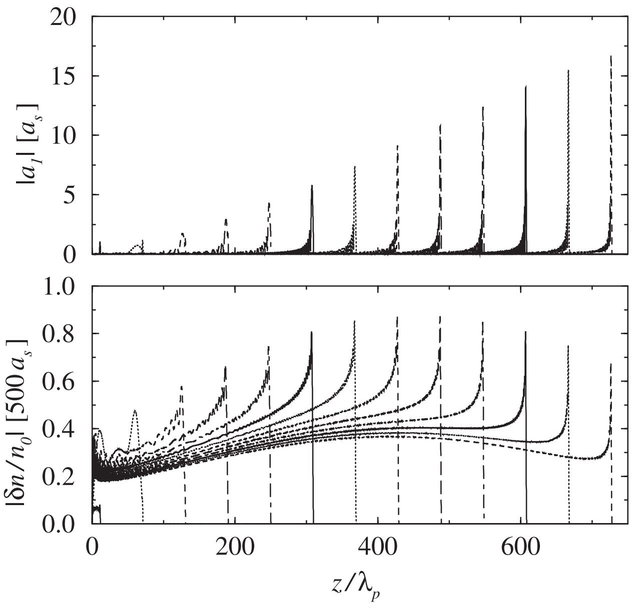

FIG. 3. Snapshots of amplitudes of probe vector potential (top) and density modulations (bottom), scaled with the maximum seed amplitude, a_s; plasma frequency $\omega_p = 4.8 \times 10^{13}$ s^{-1}, central pump frequency $\omega_0^{(0)} = 2.36 \times 10^{15}$ s^{-1}, resonant growth rate $\gamma_0 = 1.05 \times 10^{12}$ s^{-1}, seed duration 10^{-13} s, pump chirp $\alpha = 2.9 \times 10^{23}$ s^{-2}, $q = 1.9$.

The spectral distribution of the gain is evident in the Fourier spectra (Fig. 4, corresponding to the snapshots of Fig. 3), which show the amplification of successive wave number components of the scattered field (with k corresponding to frequency $\omega = ck$).

To summarize, Raman amplification using a chirped pump, as discussed in this Letter, shows interesting and useful features: First, for each frequency component of the seed, growth is limited by detuning from the resonance; overall growth of the amplitude of a short pulse is thus linear with propagation distance, not exponential, and the probe shortens self-similarly. This is superradiant scaling. Second, the amplification uses the broad bandwidth of the pump, facilitating the amplification of short pulses. Third, the gain is distributed, according to the positions where different frequency components are in resonance. This limits the plasma wave amplitude and avoids wave breaking. It also tends to suppress spontaneous scattering, pedestals, and prepulses.

We have shown that the features of superradiance: gain saturation, bandwidth broadening, and the consequent shortening of the amplified probe, which are normally associated with the nonlinear stages of resonant Raman and Compton scattering [16], are present in the linear regime of chirped pump Raman amplification.

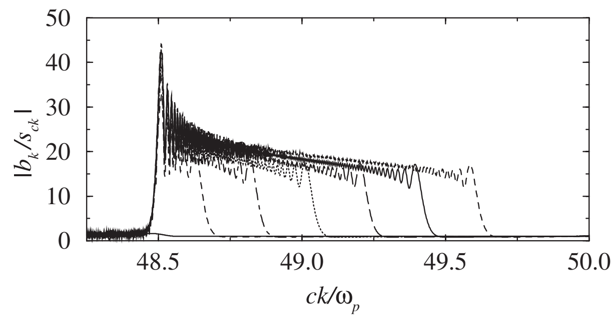

FIG. 4. Wave number spectra b_k of probe field, corresponding to the snapshots in Fig. 3 (top), normalized with seed spectrum s_ω (with $\omega = ck$).

We acknowledge the support of the Research Councils, U.K.

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