Envelope equations and conservation laws describing wakefield generation and electron acceleration

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Previous authors have proposed various envelope equations to describe the behavior of an electromagnetic pulse generating a wakefield. In general these retain second-order derivatives, the reason being that the eikonal contains the initial wave frequency. Here it is shown that if the evolution of the wave frequency is followed using ray-tracing equations, a first-order evolution equation is obtained. It can be shown with this formalism that wave action is conserved and the energy lost from the electromagnetic wave can be explicitly accounted for in terms of energy gained by the plasma. The energy balance equations suggest that an electron bunch which will extract energy efficiently from a wakefield can be at least as efficiently accelerated by direct interaction with the electromagnetic pulse. [DOI: 10.1063/1.1638753]

I. INTRODUCTION

Various envelope equations have been proposed to describe the propagation of a high amplitude laser pulse in a plasma and the way in which it evolves due to excitation of a plasma wake.[1–3] In all of these cases the equations are of second order, so that the normal objective of an envelope equation, to reduce the system to first order, is not fulfilled. The reason for keeping second-order terms is to retain essential features of the physics. The objective of this paper is to show that it is possible to construct a consistent first-order equation which can be shown to have an energy conservation law in which energy transferred to or from the electromagnetic pulse is exactly accounted for by changes in the energy of the background plasma and the electrostatic fields associated with it. The key is to use ray tracing to follow the evolution of frequency and wavenumber following the pulse. These exact values are then used in the eikonal. The usual equations for beat wave generation are shown to be in agreement with the general result. There is also a conservation law which corresponds to conservation of the number of quanta (or wave action if a classical picture is preferred) as proposed in the development of the theory of photon acceleration and deceleration.[4,5] While the idea of photon deceleration is often used to explain the loss of energy of a pulse driving a wakefield, the detailed demonstration that the energy lost by this process is exactly accounted for by the energy gain of the plasma is, as far as we are aware, new.

Turning to the problem of electron acceleration, we will show that if it is possible to generate a bunch of relativistic electrons of a length, shape and density necessary to extract energy from a wakefield with any reasonable degree of efficiency, then it should be possible to put energy directly into the electrons from the electromagnetic pulse with the same or greater efficiency. What is needed is a bunch of electrons whose density is high enough to slow the laser pulse to the speed of the electrons. If this is done then a large part of the laser energy can be transferred to the electrons. Some qualitative considerations of the effect in real three-dimensional systems indicate that it may also be possible to compress and focus the electrons. This scheme differs from other proposals for vacuum acceleration which are based on single particle trajectories in a focused laser beam[8] since an essential feature is that the electron bunch is dense enough to act like a non-neutral plasma and affect the propagation of the laser pulse. Perhaps the scheme closest to this is that of Startsev and McKinstrie[9,10] who propose ponderomotive acceleration of electrons with a laser beam slowed below the speed of light in an underdense background plasma. Here the accelerated electron bunch is supposed dense enough to slow the wave itself.

II. THE ENVELOPE EQUATIONS AND CONSERVATION LAWS

As shown in the papers on wakefield acceleration cited previously, the vector potential associated with the laser pulse obeys the wave equation

$$\left(\frac{\partial^2}{\partial t^2}-c^2\nabla^2\right)\mathbf{A}=-\omega_{\mathbf{p}}^{\mathbf{2}}\mathbf{A}, \tag{1}$$

where

$$\omega_p^2=\frac{ne^2}{\varepsilon_0 m\gamma}. \tag{2}$$

The plasma frequency is defined so as to contain the relativistic correction to the electron mass. For our purposes it will be simplest to assume the wave to be circularly polarized, so that on the short scale length of the electromagnetic wave γ is constant and the wave propagates just like a linear wave with an increased electron mass. In the case of a linearly polarized wave there are complications associated with the generation of harmonic components. These are problems we do not wish to address. For a linearly polarized wave γ is best thought of as an average value and the intensity to be such that harmonic distortions to the wave are not important. It will be convenient to take A to be normalized to be in units of mc/e, particle momentum in units of mc and the scalar potential ϕ in units of mc^2/e. Similarly, velocity, momentum and energy will be scaled to c, mc and mc^2, respectively, and distances to ct.

To see how the frequency and wavenumber vary as we follow the pulse (or different parts of the pulse if we wish to see how its shape evolves) we use the standard ray-tracing equations,[6,7]

$$\frac{d\mathbf{r}}{dt} = -\frac{\partial D/\partial \mathbf{k}}{\partial D/\partial \omega}, \tag{3}$$

$$\frac{d\mathbf{k}}{dt} = \frac{\nabla D}{\partial D/\partial \omega}, \tag{4}$$

$$\frac{d\omega}{dt} = -\frac{\partial D/\partial t}{\partial D/\partial \omega}. \tag{5}$$

The dispersion relation of the wave is $D(\mathbf{r},\mathbf{t},\mathbf{k},\omega)=0$, and in our problem

$$D = \omega^2 - \omega_p^2(\mathbf{r},t) - k^2c^2. \tag{6}$$

According to the standard expression for energy density in a wave,[7] the energy density in the electromagnetic pulse is

$$\frac{1}{2}\varepsilon_0 \frac{\partial}{\partial \omega}\left[\omega\left(1-\frac{\omega_p^2}{\omega^2}\right)\right]E^2 + \frac{1}{2\mu_0}B^2,$$

which in terms of A is

$$\frac{1}{2}A^2 \frac{m\varepsilon_0}{e^2}(\omega^2+\omega_p^2+k^2c^2) = \frac{m\varepsilon_0}{e^2}\omega^2 A^2 \tag{7}$$

(with energy in units of mc^2 if A is normalized as described above). If we take the wave to be propagating in a cold plasma then the energy flux is just the Poynting vector or, in terms of the vector potential,

$$\frac{m\varepsilon_0}{e^2}\omega \mathbf{k} c^2 A^2.$$

As would be expected the ratio of this to energy density is $\mathbf{k}\mathbf{c}^2/\omega$, the group velocity.

We now make an eikonal approximation with

$$\mathbf{A} = \mathbf{a}(\mathbf{r},t)\exp(i\psi(\mathbf{r},t)) \tag{8}$$

and

$$i\mathbf{k} = \nabla\psi,$$

$$i\omega = -\frac{\partial\psi}{\partial t}.$$

As is normal, the dependence of $\mathbf{a}$ on the space and time variables is supposed to be on longer scales than the rapid variation in the eikonal, so that second derivatives of $\mathbf{a}$ can be neglected. The point to notice here and where we differ from the authors cited previously is that we take $\mathbf{k}$ and ω to be exact values, following the pulse, and not based on the initial wave frequency. The latter procedure can be adopted, but then second-order derivatives have to be kept to get the frequency shift.

If we now put (8) into (1) in the usual way, we get

$$2c^2(\mathbf{k}\cdot\nabla)\mathbf{a} + c^2(\mathbf{a}\cdot\nabla)\mathbf{k} + 2\omega\frac{\partial \mathbf{a}}{\partial t} + \frac{\partial\omega}{\partial t}\mathbf{a} = \mathbf{0}, \tag{9}$$

which is the envelope equation we seek. We now derive some conservation laws from this. First, we multiply it by $\mathbf{a}^*$ then add the result to its complex conjugate, the result being

$$\nabla\cdot.(\mathbf{k}c^2a^2) + \frac{\partial}{\partial t}(\omega a^2) = 0,$$

where a^2 is used to denote $|\mathbf{a}|^2$. Introducing the group velocity we obtain

$$\nabla\cdot(\mathbf{v_g}\omega a^2) + \frac{\partial}{\partial t}(\omega a^2) = 0. \tag{10}$$

This is a simple evolution equation for the pulse amplitude. Since the energy density in the pulse is, as discussed above, proportional to ω^2a^2, the quantity ωa^2 can be interpreted as the density of wave quanta, or in classical terms the wave action. Equation (10) simply says that quanta are conserved, taking them to be convected with the group velocity.

Now, let us use (10) to consider energy conservation. To do this we need $\omega^2 a^2$ inside the derivatives, so we write the equation in the equivalent form

$$\nabla\cdot(\mathbf{v_g}\omega^2a^2) + \frac{\partial}{\partial t}(\omega^2a^2) = \omega a^2\left(\mathbf{v_g}.\nabla\omega + \frac{\partial\omega}{\partial t}\right)$$
$$= \omega a^2 \frac{\mathbf{d}\omega}{\mathbf{d}t}, \tag{11}$$

where d/dt is the total derivative moving with the group velocity. From (5) and (6) this is

$$\nabla\cdot(\mathbf{v_g}\omega^2a^2) + \frac{\partial}{\partial t}(\omega^2a^2) = \frac{1}{2}a^2\frac{\partial\omega_\mathbf{p}^2}{\partial t}. \tag{12}$$

From the form of this it is clear that the right-hand side (multiplied by $m\varepsilon_0/e^2$) represents the rate of energy gain (or loss if it is negative) by the electromagnetic wave. Now, to complete the demonstration that we have a self-consistent formulation, we need to show that this is the energy lost by the plasma and the slowly varying electric and magnetic fields associated with it.

Since ω_p^2 is $ne^2/\varepsilon_0 m\gamma$ we have

$$a^2\frac{m\varepsilon_0}{e^2}\frac{1}{2}\frac{\partial\omega_p^2}{\partial t} = \frac{1}{2}a^2\frac{\partial}{\partial t}\left(\frac{n}{\gamma}\right). \tag{13}$$

Now we introduce equations which describe the slow variation of the electron fluid, derived just as in standard considerations of the wakefield. Using

$$a^2=\gamma^2-p^2-1, \tag{14}$$

with p the particle momentum excluding the rapid oscillation in the electromagnetic field, we get

$$\begin{aligned} a^2\frac{1}{2}\frac{\partial}{\partial t}\left(\frac{n}{\gamma}\right)&=\frac{1}{2}\frac{\partial}{\partial t}\left(a^2\frac{n}{\gamma}\right)-\frac{1}{2}\frac{n}{\gamma}\frac{\partial}{\partial t}(a^2)\\ &=\frac{\partial}{\partial t}\left(\frac{1}{2}a^2\frac{n}{\gamma}\right)-n\left(\frac{\partial\gamma}{\partial t}\right)+\frac{n\mathbf{p}}{\gamma}.\frac{\partial\mathbf{p}}{\partial t}\\ &=\frac{\partial}{\partial t}\left(\frac{1}{2}a^2\frac{n}{\gamma}-n\gamma\right)+\gamma\frac{\partial}{\partial t}(n)+\frac{n\mathbf{p}}{\gamma}.\frac{\partial\mathbf{p}}{\partial t}. \end{aligned} \tag{15}$$

From the particle conservation equation,

$$\frac{\partial n}{\partial t}+\nabla\cdot(n\mathbf{v})=\mathbf{0},$$

the momentum equation,

$$\frac{\partial(\mathbf{p}+\mathbf{A}_0)}{\partial t}=\nabla\phi-\nabla\gamma$$

(where the reversal of signs from normal in the potentials is because of the negative electron charge) and the fact that

$$\mathbf{v}=\frac{\mathbf{p}}{\gamma},$$

we obtain

$$\begin{aligned} a^2\frac{1}{2}\frac{\partial}{\partial t}\left(\frac{n}{\gamma}\right)=&\frac{\partial}{\partial t}\left(\frac{1}{2}a^2\frac{n}{\gamma}-n\gamma\right)-\gamma\nabla\cdot\left(\frac{n\mathbf{p}}{\gamma}\right)+\frac{n\mathbf{p}}{\gamma}\cdot\nabla\phi\\ &-\frac{n\mathbf{p}}{\gamma}\cdot\nabla\gamma-\frac{n\mathbf{p}}{\gamma}\cdot\frac{\partial\mathbf{A_0}}{\partial t}, \end{aligned}$$

or

$$\begin{aligned} a^2\frac{1}{2}\frac{\partial}{\partial t}\left(\frac{n}{\gamma}\right)=&-\frac{\partial}{\partial t}\left(n\gamma-\frac{1}{2}a^2\frac{n}{\gamma}\right)-\nabla\cdot(n\mathbf{p})\\ &+\frac{n\mathbf{p}}{\gamma}\cdot\left(\nabla\phi-\frac{\partial\mathbf{A_0}}{\partial t}\right). \end{aligned}$$

Here $\mathbf{A_0}$ and ϕ are the vector and scalar potential associated with the slowly varying fields produced by the plasma. Now note that $n\mathbf{p}/\gamma\cdot(\nabla\phi-\partial\mathbf{A_0}/\partial t)$ is proportional to $\mathbf{J}\cdot\mathbf{E_0}$ where $\mathbf{J}$ is the current (assuming the ions, if there are any present, to be stationary in the laboratory frame in which we are working) and $\mathbf{E_0}$ is the electric field arising from the slowly varying potentials. So finally we obtain

$$-a^2\frac{1}{2}\frac{\partial}{\partial t}\left(\frac{n}{\gamma}\right)=\frac{\partial}{\partial t}\left(n\gamma-\frac{1}{2}a^2\frac{n}{\gamma}\right)+\nabla.(n\mathbf{p})+\frac{\partial W_{\mathbf{em}}}{\partial t}, \tag{16}$$

with $W_{\mathbf{em}}$ the energy in the slowly varying electric and magnetic fields. The left hand side here is the rate at which the electromagnetic pulse loses energy. If we look at the right-hand side, the first term is the rate of change of the particle energy density, with the part which comes from the high frequency oscillation in the electromagnetic wave subtracted off. This particle contribution to the wave energy has already been included on the left-hand side, as can be seen from (7). The second term is the divergence of the energy flux in the particles, as becomes clearer if it is written as $\nabla\cdot(n\gamma\mathbf{v})$. There is no wave contribution to the energy flux in a cold plasma. Finally we have a contribution which is the rate of change of the energy density associated with the slowly varying electric and magnetic fields produced by the plasma. So, we end up with a completely consistent description of the transfer of energy from the electromagnetic pulse to the electrons, valid for arbitrary electron density and momentum. The evolution of the laser pulse is described by the ray-tracing equations, supplemented by the condition that the integral of ωa^2 over any three-dimensional volume carried along with the rays is constant, while the rate of transfer of energy from the wave to the electrons is determined by the value of $\omega a^2(d\omega/dt)$ along the ray path. To determine the pulse evolution each point of the envelope should be followed in position and wavenumber space (the frequency does not need to be followed separately but can be obtained from the dispersion relation). In this way the change in shape due to differences in group velocity and in photon deceleration in different parts of the pulse can be followed. As pointed out by Sprangle *et al.*,[11] wavenumber dispersion may also be important. This can be taken into account within the present geometrical optics formulation by regarding the pulse as a superposition of different wavenumber components. This procedure, described by Mendonça,[5] is valid as long as the time scale associated with the pulse is long compared with that associated with the optical frequency oscillations. Each wavenumber component will propagate according to the above description and the energy transfer just be a superposition of the effects of these components. The spectrum of wavenumber components is determined by the Fourier transform of the initial pulse shape. If, as is common, this is taken to be a Gaussian then the wavenumber spectrum is also Gaussian with a width inversely proportional to the spatial extent of the pulse. This applies, of course, to both the longitudinal and transverse components of the spectrum. Finally we should say that as the system evolves it is possible that steep gradients or local focusing or formation of caustics by the rays may invalidate the geometrical optics approximation.

III. APPLICATION TO THE WAKEFIELD PROBLEM

Now we relate this to the one-dimensional quasistatic wake equations, to demonstrate its consistency with familiar results. In this case all the plasma properties are assumed to be a function of $z-t$ (in suitably scaled units) so

$$\frac{\partial}{\partial t}\rightarrow-\frac{\partial}{\partial z},$$

and the general result becomes

$$a^2\frac{1}{2}\frac{\partial}{\partial z}\left(\frac{n}{\gamma}\right)=-\frac{\partial}{\partial z}\left(n\gamma-\frac{1}{2}a^2\frac{n}{\gamma}\right)+\frac{\partial}{\partial z}(np)-\frac{\partial}{\partial z}\left(\frac{1}{2}E^2\right), \tag{17}$$

with E an appropriately normalized longitudinal electric field. Now, according to, for example, Sprangle, Esary and Ting,[1] the various plasma quantities are related to the potential in this approximation through

$$n=\frac{1+a^2+(1+\phi)^2}{2(1+\phi)^2},$$
$$\gamma=\frac{1+a^2+(1+\phi)^2}{2(1+\phi)}, \tag{18}$$
$$\omega_p^2=\omega_{p0}^2\frac{n}{\gamma}=\omega_{p0}^2\frac{1}{1+\phi},$$

where ω_{p0}^2 is calculated from the unperturbed plasma density and n is normalized to this density. From this we can obtain, since

$$p^2=\gamma^2-1-a^2,$$

the relations

$$p=\frac{1+a^2-(1+\phi)^2}{2(1+\phi)}$$

and

$$\gamma-p=1+\phi.$$

Using these, the conservation relation becomes

$$\frac{1}{2}\frac{\omega_{p0}^2}{c^2}a^2\frac{\partial}{\partial z}\left(\frac{1}{1+\phi}\right)=-\frac{\omega_{p0}^2}{c^2}\frac{\partial}{\partial z}\left[\frac{1+(1+\phi)^2}{2(1+\phi)}\right]-\frac{1}{2}\frac{\partial}{\partial z}\left(\frac{\partial\phi}{\partial z}\right)^2. \tag{19}$$

Rearranging and cancelling out a factor $\partial\phi/\partial z$ gives a result equivalent to Poisson's equation in the quasistatic case. Our conservation relation is thus entirely consistent with the well-known quasistatic wakefield equations.

It is interesting to discuss the application of this approach to some recent work by Sprangle *et al.*[11] These authors obtain a higher order envelope equation and discuss the evolution of the electromagnetic pulse shape. Higher order terms are needed to include the frequency evolution, since their eikonal is calculated with a constant frequency. In our approach, the variation in frequency, wavenumber and group velocity is calculated across the pulse profile by use of the ray-tracing equations, while the amplitude variation can be found from (10). If we consider a simple one-dimensional approximation where the plasma density can be assumed to be a function of $z-ct$, then

$$\frac{dk}{dt}=-\frac{1}{2\omega}\frac{\partial}{\partial z}\omega_p^2,$$
$$\frac{d\omega}{dt}=-\frac{c}{2\omega}\frac{\partial}{\partial z}\omega_p^2, \tag{20}$$
$$\frac{dv_g}{dt}=-\frac{c}{2\omega}(c-v_g)\frac{\partial}{\partial z}\omega_p^2.$$

At the leading edge of the pulse (assumed to be moving in the positive direction), the gradient of ω_p^2 is positive, as the pulse produces an initial density cavity, and so the frequency, wavenumber and group velocity all decrease. The peak of the pulse moves backwards while, from (10) its amplitude (at least expressed in terms of the vector potential) increases. This is qualitatively in agreement with the results of Sprangle *et al.*[11] It is also qualitatively consistent with some recent results obtained by Reitsma,[12] also using a higher order envelope equation.

IV. DISCUSSION AND FURTHER COMMENTS ON ELECTRON ACCELERATION

To emphasize once more the difference between our approach and others which use a higher order envelope equation, the latter use a known frequency and wavenumber in the eikonal. In our case we determine the local wavenumber and frequency at each point using the ray-tracing equations in standard form. The wave amplitude can then be determined from a first order equation which, written in different ways, can be interpreted as a conservation equation for wave action (or density of wave quanta) or as an energy equation. In the energy equation the energy gained or lost by the electromagnetic pulse has been shown to be exactly balanced by the change in energy of the plasma particles and the slow part of the electromagnetic field. As far as we are aware this is the first time that the energy transfer between the laser pulse and wake has been considered in such detail and in a general setting in which a quasistatic wake is not assumed.

Considering the wakefield problem again, if the plasma can be regarded as having a stationary profile which is a function of $z-vt$, with $v\sim c$, then the rate of energy loss to the wake is $-(1/2)(a^2c)(\partial/\partial z)\,\omega_p^2$, integrated over the pulse profile. Even if an electron bunch can be optimally placed to extract all the energy from the wave, this places an upper bound on the rate at which it can gain energy. Suppose that instead we placed the electromagnetic pulse on the trailing edge of a relativistic electron bunch, adjusting the density and frequency so that the pulse and the bunch traveled together. On the trailing edge, the density gradient would be such as to produce wave energy absorption by the electrons, and if the density and scale length of the bunch were comparable to the density used in the wakefield scheme and the wakefield wavelength, then there would be a comparable rate of energy absorption, guaranteed to go entirely to the relativistic electron bunch.

We can see in more detail what would happen if we consider first a one-dimensional problem, taking an electron bunch moving rigidly with speed V and changing to the rest frame of this bunch. In this frame the point where the laser pulse is slowed to the beam velocity in the laboratory frame corresponds to the critical density at which the pulse, Doppler shifted to a lower frequency, is reflected with unchanged frequency. Transforming back to the laboratory frame, this corresponds to the laser pulse overtaking the electrons, mov-

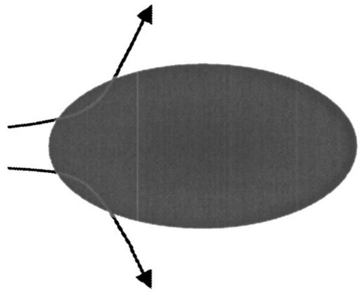

FIG. 1. Schematic of ray paths for the wave overtaking a plasma bunch. The outwards refraction will produce a focusing effect on the bunch.

ing up the density gradient until its speed becomes equal to that of the electrons, then coming back down the density gradient and eventually being reflected. The ratio of incident to final frequency in this frame is

$$\frac{c-V}{c+V},$$

a small number if $V \sim c$. In this situation we would expect almost complete transfer of energy from the laser to the electrons. Note that the electron density needed in the laboratory frame is that which slows the group velocity to the electron bunch velocity, not the critical density in this frame which would be much higher.

In practice important questions would arise as to whether the electron bunch could be focused and compressed in three dimensions. To answer such questions in detail will need a detailed study of the dynamics of the electron bunch, but if we consider the qualitative nature of the ray paths, then we expect something like the behavior sketched in Fig. 1. Rays hitting the edges of the bunch are refracted outwards, so there will be an inwards force tending to produce axial bunching. We might imagine that with a suitably tailored laser pulse it would be possible to compress and focus the electron bunch in a smooth way, the problem being rather akin to the old one of producing smooth compression of an imploding shell.

V. CONCLUSIONS

We have shown that a consistent treatment of laser pulse propagation in an underdense plasma can be obtained from standard ray tracing techniques, supplemented by a condition which expresses conservation of wave action along the ray path and which fixes the amplitude variation. An equation for energy transfer from the pulse to the plasma is obtained and it is shown to be consistent with a detailed consideration of the electron dynamics. Applying this to the wakefield, the conclusion is that the rate at which energy is transferred from the pulse to the wake, which of course puts an upper bound on the rate at which energy can be transferred to an accelerated electron bunch, depends on the plasma gradient produced by the pulse. To extract energy efficiently from the wake needs an electron bunch of a length comparable to or shorter than the wavelength of the wake, and of a density comparable to that of the background plasma. Smaller numbers of electrons can, of course be accelerated, but this makes very inefficient use of the laser energy. If a sufficiently short and dense electron bunch can be produced in a vacuum and a laser pulse launched just behind it, then the laser pulse will be confined to the trailing edge of the bunch where it will give up energy at a rate comparable to the maximum rate at which energy can be transferred in the wakefield scheme. While further work needs to be done on the details of the electron dynamics, it seems possible that this scheme could be used to produce acceleration and focusing of an electron bunch in a simpler way than the wakefield.

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