ELSEVIER

Nuclear Instruments and Methods in Physics Research A 445 (2000) 72–76

NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH Section A

www.elsevier.nl/locate/nima

SACSE in a FEL amplifier with energy spread

B.W.J. McNeil*, G.R.M. Robb, D.A. Jaroszynski

Department of Physics & Applied Physics, John Anderson Building, University of Strathclyde, 107, Rottenrow, Glasgow G4 0NG, Scotland, UK

Abstract

An analysis of the Free Electron Laser (FEL) operating in the non-linear high-gain regime and including the effects of Coherent Spontaneous Emission (CSE) and electron energy spread is presented. The results from this model are compared with previous work which neglected the effects of CSE. The results show that CSE can significantly reduce the start-up time and enhance the generation of high intensity, short, superradiant radiation pulses in a poor-quality electron pulse. © 2000 Elsevier Science B.V. All rights reserved.

PACS: 41.60.C

1. Introduction

The interaction between the electrons and radiation in an FEL amplifier may be initiated by 'shot noise' emission, due to the random position of electrons within the electron pulse, or by a small resonant seed field injected into the interaction region. When the radiation develops in an amplifier from shot noise, the process has been called self-amplified spontaneous emission (SASE) [1]. FELs operating in the SASE regime at infrared wavelengths have been the subject of recent experimental investigation [2–5] with a view to providing a proof-of-principle verification of SASE operation for potential VUV/X-ray FELs [6,7].

Another spontaneous emission process that can initiate the FEL interaction arises from the initial current profile, or 'shape' of the electron pulse. This has been called 'coherent spontaneous emission' (CSE) [8,9] and has been the subject of a number of recent studies, both theoretical and experimental [8–13]. CSE with intensities several orders of magnitude greater than that due to shot noise alone may occur when the current profile of the electron pulse changes significantly on the scale of a radiation wavelength. An analysis of the amplification of CSE was originally carried out for the Cherenkov maser [12], its interaction being described by a set of equations that are very similar to those describing the FEL. The amplification of CSE in the FEL has been shown numerically to be superradiant in nature, yielding higher peak powers than SASE alone and reducing shot-to-shot fluctuations [13]. This process has been called self-amplified coherent spontaneous emission (SACSE) [13] and to date has been modelled only with monoenergetic electron pulses.

Here, we present an analysis of SACSE including the effects of energy spread in the electron pulse.

* Corresponding author. Tel.: + 44-141-548-4727; fax: + 44-141-552-2891.

E-mail address: cabs46@strath.ac.uk (B.W.J. Mc Neil).

PII: S0168-9002(00)00116-9

We make a comparison with a previous method of analysis where CSE is absent and show that, for an electron pulse with a rectangular current distribution, SACSE may significantly enhance radiation powers.

2. Theory

To date, most analytical studies of the FEL operating with electron pulses have involved the averaging and discretisation of the wave equation over an interval equal to or greater than a radiation period [14]. This restricts the description of the field evolution to situations where the current profile of the electron pulse changes negligibly on the scale of the radiation period. In Ref. [13] no such average was performed allowing for a more accurate description of the electromagnetic field evolution.

As with [13], we consider the interaction between a pulse of electrons with an initial mean energy $\langle\gamma\rangle_0 mc^2$ which is resonant with a plane, circularly polarised radiation field in a helical wiggler FEL. The field evolution is described by Maxwell's wave equation and the electron trajectories, which determine the source current for the radiation field, are governed by the Lorentz force equation. In terms of the scaled variables and assumptions of Ref. [13], the coupled Maxwell/Lorentz equations may be written

$$\frac{\mathrm{d}\bar{z}_{1_j}}{\mathrm{d}\bar{z}} = 2\rho p_j \tag{1}$$

$$\frac{\mathrm{d}p_j}{\mathrm{d}\bar{z}} = -\left(A(\bar{z},\bar{z}_{1_j})\exp\left(\mathrm{i}\frac{\bar{z}_{1_j}}{2\rho}\right) + \text{c.c.}\right) \tag{2}$$

$$\left(\frac{\partial}{\partial\bar{z}} + \frac{\partial}{\partial\bar{z}_1}\right)A(\bar{z},\bar{z}_1) = \frac{1}{\bar{n}_{||}}\sum_{j=1}^{N}\chi_j\delta(\bar{z}_1 - \bar{z}_{1_j}) \times\exp\left(-\mathrm{i}\frac{\bar{z}_1}{2\rho}\right). \tag{3}$$

An energy spread may be modelled by a distribution of the scaled momentum parameter $p_j = (\gamma_j - \gamma_\mathrm{r})/\rho\gamma_\mathrm{r}$, where γ_r is the resonant energy for the radiation field. In what follows, we assume the electrons are distributed uniformly in scaled position and momentum space over the intervals $\bar{z}_1 \in [0,\bar{l}_\mathrm{e}]$ and $p \in [\delta - \sigma_p, \delta + \sigma_p]$ where $\bar{l}_\mathrm{e}$ is the scaled length of the electron pulse, $\delta = (\langle\gamma\rangle_0 - \gamma_\mathrm{r})/\rho\gamma_\mathrm{r}$ is the mean detuning of the electrons from resonance and σ_p is the half-width of the scaled momentum distribution. Note that the effects of electron-beam emittance may be modelled in an identical way by introducing an effective spread in the resonant energies γ_r of the electrons as described in Ref. [15].

The scaled equations (1) and (3) may be solved using the finite element method [16,17] to describe the field. This method allows the effects of CSE and SACSE to be modelled numerically. Note that a linear analysis of an equivalent set of equations has been given in Ref. [11]. An almost identical set of equations and solution method have been derived for the Cherenkov maser interaction in which SACSE has also been predicted to occur [12]. An alternative numerical method to that presented here is given in Ref. [10] where a Fourier analysis is used.

To the authors' knowledge, apart from [10,12,13] previous numerical analyses have averaged the wave equation (3) over at least one radiation period (e.g. as in Ref. [14]), here corresponding to an interval in $\bar{z}_1$ of $[0,4\pi\rho]$, within which the charge weightings χ_j are assumed equal. In this case, the field is an averaged variable, $A(\bar{z},\bar{z}_1) \to \bar{A}(\bar{z},\bar{z}_1)$, and the source term of Eq. (3) reduces to

$$\frac{1}{\bar{n}_{||}}\sum_{j=1}^{N}\chi_j\delta(\bar{z}_1 - \bar{z}_{1_j})\exp\left(-\mathrm{i}\frac{\bar{z}_1}{2\rho}\right)$$

$$\to \frac{\chi(\bar{z}_1)}{N'}\sum_{j=1}^{N'}\exp\left(-\mathrm{i}\frac{\bar{z}_{1_j}}{2\rho}\right)$$

where the sum $\sum_{j=1}^{N'}$ is over the electrons contained within the periodic interval of $\bar{z}_1$ centred at the positions where the radiation field and electron pulse has been sampled and averaged, and $\chi(\bar{z}_1)$ is the electron charge weighting function evaluated at the averaging positions. Although this 'averaged model' can describe some aspects of the pulsed, self consistent interaction of radiation and electrons such as superradiance, it cannot describe the effects of SACSE.

3. Results

The significant influence of CSE on the FEL interaction is now shown by solving both the averaged and unaveraged equations for a rectangular electron pulse shape initially distributed over the interval $0 < \bar{z}_1 < \bar{l}_e$ where $\bar{l}_e = 15$ is the scaled length of the pulse. This pulse is assumed to have a uniform scaled momentum distribution of half-width $\sigma_p = 1.5$ centred at the resonant energy so that $\delta = 0$. Such a uniform momentum space distribution corresponds to charge weightings of $\chi_j = 1\ \forall j$.

It would be expected that the effects of energy spread would have no significant effect upon the CSE process as the radiation emitted in this way depends primarily upon the current distribution, or 'shape', of the electron pulse. This has been confirmed, with initial CSE radiation fields being independent of the electrons' energy distribution. The energy distribution has an effect on CSE only in that the shape of the pulse may change due to the differential electron velocities.

In the absence of shot-noise, the averaged model is only exponentially unstable if the electrons interact with a non-zero field input at the beginning of the interaction region, whereas the unaveraged model may amplify any initial field and, in addition, the self-generated CSE. We assume an initial scaled input field of $A_0 = A(\bar{z} = 0, \bar{z}_1) = 10^{-3}$ for both averaged and unaveraged interactions modelled here.

Using the linear analysis of Ref. [15], it is easily shown that there is a threshold in the energy spread above which there can be no exponential growth of the field in the steady-state region of the electron pulse defined by the range $\bar{z} < \bar{z}_1 < \bar{l}_e$. This threshold occurs at $\sigma_p = (27/4)^{1/6} \approx 1.37$, (for $\delta = 0$) so that for the spread used here no exponential steady-state evolution of the field occurs. (Note, however, that a maximum in the gain occurs for $\delta = \sigma_p$ which is not considered here.)

Using the above parameters, and a FEL parameter of $\rho = 0.01$, in Fig. 1 we plot the scaled radiation intensity $|A|^2$ as a function of $\bar{z}_1$ for a scaled interaction length of $\bar{z} = 15$ obtained from a numerical solution of the unaveraged equations. It can be seen that SACSE produces a high-intensity spike of radiation. The CSE within the slippage region acts as a strong seed field from which the high-intensity spike of radiation quickly develops as it is amplified on propagating through the electron pulse. Comparison with a solution of the averaged model solved using identical parameters is shown Fig. 2. The enhanced radiation emission when CSE effects are included in the model clearly demonstrates the importance of SACSE in the

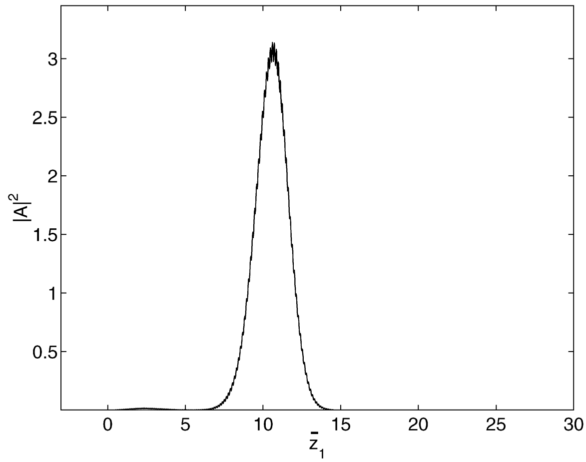

Fig. 1. The scaled field intensity $|\bar{A}|^2$ of the SACSE model plotted as a function of $\bar{z}_1$ when $\bar{z} = 15$, for a rectangular electron pulse current profile of scaled length $\bar{l}_e = 15$ and $\rho = 0.01$

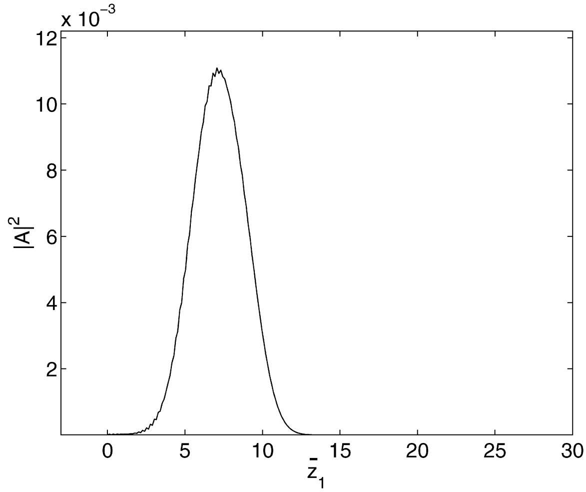

Fig. 2. The scaled field intensities $|A|^2$ for the averaged model, plotted as functions of $\bar{z}_1$ when $\bar{z} = 15$, for a rectangular electron pulse current profile of scaled length $\bar{l}_e = 15$ and $\rho = 0.01$.

non-linear evolution of the radiation field. It can be shown that the intensity spikes, both with and without the effects of CSE, are superradiant in nature, their intensity being proportional to n_e^2, where n_e is the electron density of the electron pulse.

Although in the absence of SACSE an intense spike does eventually evolve, its evolution is significantly retarded. To the authors knowledge, the superradiant emission from an electron pulse with an energy spread above the threshold limit for growth (in the steady state) has not been reported before in either the SACSE *or* the averaged model.

In order to investigate the energy emitted by the electron pulse, we define the scaled radiation energy

$$E(\bar{z}) = \int_{-\infty}^{+\infty} |A(\bar{z},\bar{z}_1)|^2 \, d\bar{z}_1 .$$

This is plotted in Fig. 3 for both the SACSE and averaged model examples of Figs. 1 and 2. It is seen that, for this example, the energy emitted by the electron pulse in the SACSE model is approximately 200 times that of the averaged model at the end of the interaction region. Once a self-similar type superradiant pulse has been established, the scaled energy E takes on the non-exponential form

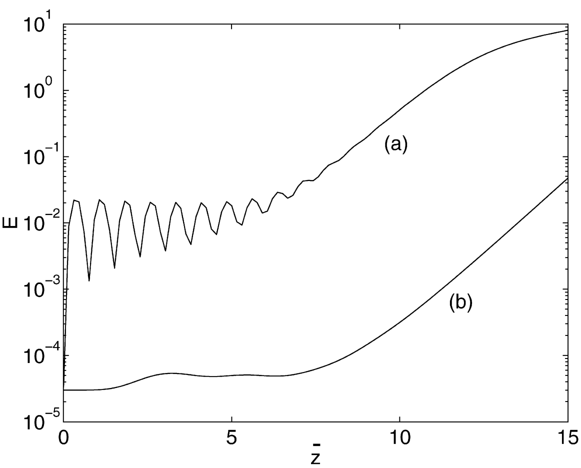

Fig. 3. The scaled radiation energies $E(\bar{z})$ for (a) the SACSE model, and (b) the averaged model, plotted as functions of $\bar{z}$, for a rectangular electron pulse current profile of scaled length $\bar{l}_e = 15$. In both cases $\rho = 0.01$.

$E(\bar{z}) \propto \bar{z}^{3/2}$. The transition to this scaling can be seen for the SACSE solution for $\bar{z} > 12$. The radiation emitted in the averaged model solution has not attained a self-similar solution and the superradiant scaling of the scaled energy has not yet developed.

4. Conclusions

We have presented a one-dimensional analysis of a high-gain FEL amplifier in the Compton limit, operating with a rectangular shaped electron current profile, and taking into account the effect of CSE and electron energy spread on the electron-radiation field interaction. As CSE depends primarily upon the shape of the electron pulse, electron pulse quality has little effect upon its generation. It was shown for a specific example that self-amplification of CSE (SACSE) from such a poor-quality electron pulse generates a strong superradiant pulse of radiation. The peak intensity and the energy emitted were both significantly greater than that where CSE effects are not included in the model.

These results are important for FEL experiments where variations of the electron pulse current on the scale of the radiation wavelength are significant and where electron pulse quality would previously have been thought to prohibit any radiative instability for the resonant interaction ($\delta = 0$).

Acknowledgements

The authors would like to thank the EPSRC and the Royal Society of Edinburgh for support of BMcN and GRMR, respectively.

References

[1] R. Bonifacio, C. Pelligrini, L.M. Narducci, Opt. Commun. 50 (1984) 373.
[2] M. Hogan et al., Phys. Rev. Lett. 80 (1998) 289.
[3] D.C. Nguyen et al., Phys. Rev. Lett. 81 (1998) 810.
[4] M. Hogan et al., Phys. Rev. Lett. 81 (1998) 4867.
[5] D. Bocek et al., Nucl. Instr. and Meth. A 375 (1996) 13.
[6] J. Rossbach, Nucl. Instr. and Meth. A 375 (1996) 269.

[7] R. Tatchyn, Nucl. Instr. and Meth. A 375 (1996) 274.
[8] A. Doria et al., IEEE J. Quantum Electron. 29 (1993) 1428.
[9] D.A. Jaroszynski et al., Phys. Rev. Lett. 71 (1993) 3798.
[10] N. Piovella, in: R. Bonifacio, W.A. Barletta (Eds.), Towards X-Ray Free Electron Lasers, AIP Conference Proceedings No. 413, AIP, New York, 1997, p. 205.
[11] S. Krinsky, Phys. Rev. E 59 (1999) 1171.
[12] B.W.J. McNeil, G.R.M. Robb, D.A. Jaroszynski, Opt. Commun. 163 (1999) 203.
[13] B.W.J. McNeil, G.R.M. Robb, D.A. Jaroszynski, Opt. Commun. 165 (1999) 65.
[14] R. Bonifacio, B.W.J. McNeil, P. Pierini, Phys. Rev. A 40 (1989) 4467.
[15] R. Bonifacio, L. De Salvo Souza, B.W.J. McNeil, Opt. Commun. 93 (1992) 179.
[16] E.A. Huebner, E.A. Thornton, T.G. Byrom, The Finite Element Method for Engineers, Wiley, New York, 1995.
[17] C. Johnson, Numerical Solution of Partial Differential Equations by the Finite Element Method, Cambridge University Press, Cambridge, 1995.