# Efficiency and Energy Spread in Laser-Wakefield Acceleration

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The theoretical limits on efficiency and energy spread of the laser-wakefield accelerator are investigated using a one-dimensional model. Modifications, both of the wakefield due to the electron bunch, and of the laser pulse shape due to the varying permittivity of the plasma, are described self-consistently. It is found that a short laser pulse gives a higher efficiency than a long laser pulse with the same initial energy. Energy spread can be minimized by optimizing bunch length and bunch charge such that the variation of the accelerating force along the length of the bunch is minimized. An inherent trade-off between energy spread and efficiency exists.

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In recent years, acceleration of charged particles in high-amplitude electrostatic waves [1] driven by relativistic electron beams [2] or powerful laser pulses [3,4] propagating in plasma has been demonstrated experimentally. The advantage of plasma-based methods of particle acceleration is that plasma can sustain accelerating fields 3 to 4 orders of magnitude higher than the maximum accelerating fields of conventional radio frequency accelerating cavities [5]. However, a high accelerating field is not the only requirement for realizing a practical high energy accelerator. Firstly, the accelerating field should act on the particles for a duration sufficient to obtain a high energy. Secondly, many applications require a good beam quality, i.e., a small emittance and/or a small energy spread [6]. Finally, a reasonable fraction of the energy of the plasma wave driver should ideally be transferred to the particle beam. In this Letter we discuss these requirements and propose new strategies to reconcile conflicting demands for the laser-wakefield accelerator, which is considered the most promising scheme for controlled plasma-based acceleration of electrons.

The ponderomotive pressure of an intense laser pulse in plasma displaces electrons, while ions are effectively immobile on the short time scale of the laser-plasma interaction [7]. The Coulomb field produced by charge separation drives a collective electron oscillation in the wake of the laser pulse, as shown schematically in Fig. 1. To drive the wakefield effectively, the length of the pulse should be half a plasma wavelength or less, otherwise energy transferred to the wake by the front of the pulse is recovered by the rear part. An electron injected into an accelerating segment of the wakefield gains energy from it. As the wake travels at the laser pulse group velocity, $v_g < c$, the electron will slip forward in the wave and eventually reach a maximum of the wake potential. This effect, known as dephasing, limits the electron energy gain [8]. Furthermore, the variation of the accelerating field along the longitudinal coordinate causes energy spread because the bunch has a finite length [9]. If the bunch length is one full plasma wavelength or more, then the electrons sample all phases of the wave, which results in a very large energy spread.

An important consequence of the presence of the electron bunch in the plasma is that it causes a modification of the wakefield, as illustrated in Fig. 1. This effect, also known in conventional accelerators, is called beam loading [9,10] and it limits the amount of charge that can be accelerated on a given laser wakefield, because electrons extract energy from the wake by producing their own wake which (partly) cancels that produced by the laser pulse. In addition, to achieve efficient acceleration, the laser pulse should transfer a considerable fraction of its energy to the wake during the dephasing time of the bunch. However, this necessarily leads to a spectral and temporal deformation and eventual breakup of the laser pulse [11]. In this Letter we present the first analysis, albeit in a one-dimensional geometry, of the coupled evolution of the laser pulse, the wakefield, and the electron bunch. The one-

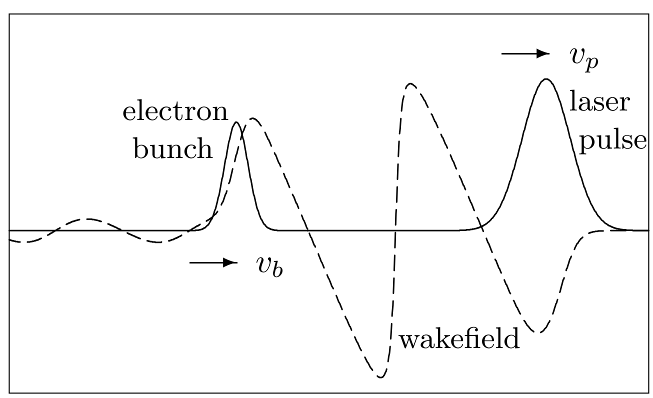


FIG. 1. Sketch of laser-wakefield acceleration. A laser pulse with velocity $v_p$ and an electron bunch with velocity $v_b$ exchange energy through the wakefield (dashed line) as they propagate in a plasma.

 

dimensional approximation limits the validity of our analysis to laser pulses and electron bunches with a typical width that is large compared to the plasma wavelength.

For our analysis we use a fully self-consistent model, which we describe here, starting with the laser pulse dynamics. The changes to the pulse are due to the local spatiotemporal refractive index changes caused by the density modulation of the wake. The evolution can be effectively followed with photon kinetic theory [12] in which the pulse is regarded as a distribution of quasiphotons with conjugate canonical space and wave number coordinates. This distribution is obtained from a Wigner-Moyal transformation of the complex electric field of the laser pulse. This approach allows the effects of refractive index variation over the length of the pulse to be described through the spatial distribution, as well as the effect of dispersion due to the wave number spread. The evolution of quasiphotons is modeled with the ray-tracing equations

$$\frac{d\zeta}{dt} = \frac{\partial \mathcal{H}_{ph}}{\partial k}, \qquad \frac{dk}{dt} = -\frac{\partial \mathcal{H}_{ph}}{\partial \zeta},$$

which are derived from the quasiphoton Hamiltonian

$$\mathcal{H}_{ph} = \omega - ck,$$

where $\zeta = z - ct$ denotes the coordinate moving at the speed of light and $\omega$ and $k$ the quasiphoton frequency and wave number, respectively. The evolution of a quasiphoton represents the collective behavior of a large number of photons in the plasma rather than the path of a single photon. The photon kinetic method is valid in underdense plasma, where the length and time scales of the plasma response are much longer than the optical period and wavelength. The dynamics of bunch electrons are described with the equations of motion

$$\frac{d\zeta}{dt} = \frac{\partial \mathcal{H}_{el}}{\partial p}, \qquad \frac{dp}{dt} = -\frac{\partial \mathcal{H}_{el}}{\partial \zeta},$$

obtained from the electron Hamiltonian

$$\mathcal{H}_{el} = mc^2(\gamma - \phi) - cp,$$

where $\zeta$ and $p$ are the conjugate canonical electron position and momentum coordinates, and $\gamma$ denotes the Lorentz factor. Both the electron and quasiphoton dynamics are governed by the scalar potential $\Phi$ (dimensionless form $\phi = e\Phi/mc^2$). The frequency of quasiphotons obeys a local dispersion relation

$$\omega^2 = c^2k^2 + \Omega_p^2, \qquad \Omega_p^2(\zeta) = \frac{c^2k_p^2}{1+\phi},$$

where $k_p^2 = 4\pi n_p e^2/mc^2$ and $n_p$ is the unperturbed plasma density. A self-consistent description is given by coupling with the quasistatic plasma electron fluid equation [7] for $\phi$,

$$\frac{\partial^2\phi}{\partial\zeta^2} = k_p^2\left(\frac{1 + a^2 - (1+\phi)^2}{2(1+\phi)^2} + \frac{n_b}{n_p}\right),$$

where circular laser polarization has been assumed. The wakefield source terms are $n_b \propto \int f_b dp$, obtained from the bunch electron distribution $f_b(\zeta, p)$ and $a^2 = e^2A^2/m^2c^4 \propto \int(f_p/\omega)dk$, obtained from the laser pulse quasiphoton distribution $f_p(\zeta, k)$, where $\vec{A}$ denotes the vector potential. The quasistatic approximation is valid as long as bunch electrons and quasiphotons move much slower than plasma electrons in the frame moving at the speed of light.

We have written a one-dimensional simulation code in which both the laser pulse and the electron bunch are treated as collections of finite-size macroparticles. At each time step, $a^2$ and $n_b$ are calculated on a spatial grid with standard particle-in-cell methods. Subsequently, the quasistatic wakefield equation is solved on the grid with a finite-difference method. Finally, the particle coordinates are moved forward in time, via the ray-tracing equations and the electron equations of motion. The initial quasiphoton distribution [12] is taken to represent a bandwidth-limited Gaussian pulse, with the distribution $f_p(\zeta, k) \propto \exp[-\zeta^2/L_l^2 - (k - k_0)^2L_l^2]$, where $L_l$ is the laser pulse length and $k_0$ the central wavenumber. For the electron bunch we choose a similar form $f_b(\zeta, p) \propto \exp[-(\zeta - \zeta_b)^2/L_b^2 - (p - p_0)^2/\Delta^2]$, where $\zeta_b$ is the injection phase of the bunch and $L_b$ the bunch length, while $p_0$ is the average momentum and $\Delta$ a measure of the initial momentum spread. To simplify matters a bit, we consider the limit of an initially monoenergetic ($\Delta \rightarrow 0$), ultrarelativistic ($p_0/mc \gg k_0/k_p$) bunch, for which $n_b \propto \exp[-(\zeta - \zeta_b)^2/L_b^2]$ is static. Other variables are the initial peak value $a_0^2$ of $a^2$, the initial peak value $n_{b0}$ of $n_b$, and the value of $k_0/k_p$.

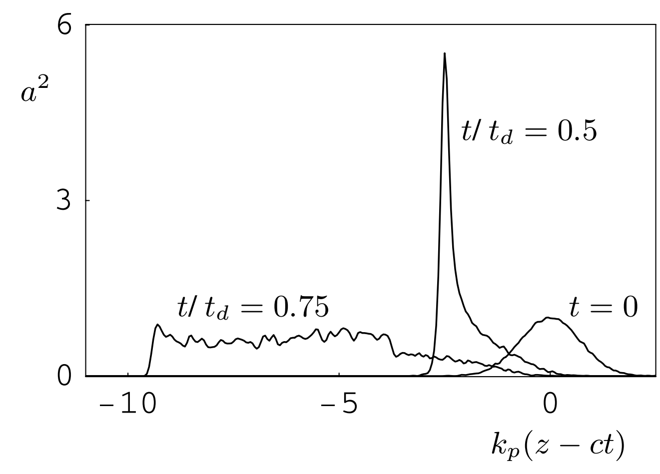


FIG. 2. Pulse deformation. Three snapshots of $a^2$ for a laser pulse with $a_0^2 = 1$, $k_pL_l = 1$, $k_0/k_p = 50$. See text for definition of dephasing time $t_d$.

Now let us discuss the mechanisms of laser pulse energy depletion and pulse deformation [11,13]. As the number of quasiphotons, $\int f_p dk d\zeta$, is conserved, a drop in frequency corresponds to a loss in energy to the wake. The pulse energy loss rate is proportional to $E_0^2$, where $E_0$ is the amplitude of the dimensionless accelerating wakefield $E = (1/k_p)\partial\phi/\partial\zeta$. The quantity $a^2$ that drives the wakefield is proportional to $\int (f_p/\omega)dk$, and therefore it increases as the frequency drops. Feedback from the wake leads to an explosive instability, because an increase of $a^2$ causes $E_0$ to grow and thus speeds up the energy depletion. However, a reduction in quasiphoton frequency not only leads to an increase of $a^2$, but also to a reduction of group velocity. This leads to pulse deformation, as the segment of the pulse where energy depletion is greatest slips backwards with respect to the rest of the pulse. Eventually, pulse deformation leads to a decrease of $E_0$ as the pulse spreads out and $a^2$ decreases. As an example, simulation results for $a_0^2 = 1$, $k_p L_l = 1$, $k_0/k_p = 50$ are given in Fig. 2. This figure shows snapshots of $a^2$ at $t/t_d = 0$, 0.5, and 0.75, where $t_d$ denotes the dephasing time defined below. Increase and subsequent decrease of $a^2$, as well as pulse deformation, are clearly visible.

The efficiency, defined as the fraction of energy transferred from the laser pulse to the electron bunch, depends on the ratio of pulse energy depletion and electron dephasing time scales. The usual definition [14] of the dephasing time $t_d$ is the time required for a relativistic electron to slip over half a plasma wavelength, $ct_d = 2\pi k_0^2/k_p^3$. In the linear regime ($E_0 \ll 1$), the efficiency is low, because only a small fraction of the pulse energy is transferred to the wake on the dephasing time scale. Higher efficiency is possible in the nonlinear regime ($E_0 \simeq 1$), as a larger fraction of the pulse energy is transferred to the wake. This is illustrated in Fig. 3, in which a comparison is made between laser pulses with the same initial energy ($a_0^2 k_p L_l = 1$) and different pulse lengths ($k_p L_l$ between 0.2 and 1.5). This Figure shows the time $t_1$ at which the laser pulse has lost half of its initial energy, the time $t_2$ at which the electron bunch reaches its maximum energy, and the fraction $\eta$ of energy transferred at $t = t_2$. This fraction is calculated in the theoretical limit $L_b \to 0$, while the values of injection phase $\zeta_b$ and bunch charge are optimized to maximize $\eta$. As expected, shorter pulses give higher efficiency, because they transfer their energy faster than long pulses.

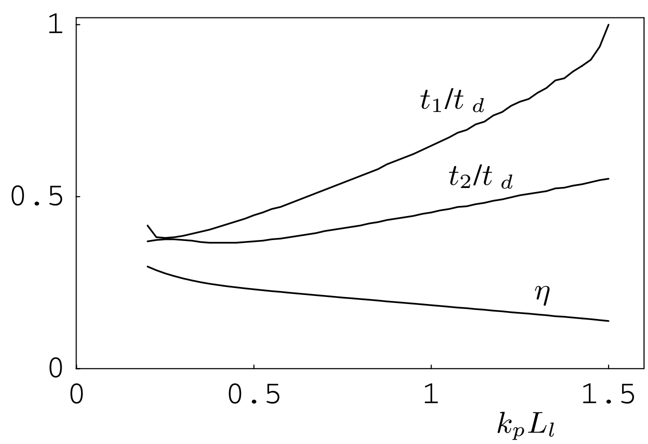


FIG. 3. Dependence of efficiency on laser pulse length. Plots of time $t_1$ at which the laser has lost half of its energy, time $t_2$ at which the electron bunch has reached its maximum energy, and efficiency $\eta$, all as functions of pulse length at constant $a_0^2 k_p L_l = 1$. The bunch charge and injection phase have been varied to maximize $\eta$ in limit of zero bunch length.

Optimizing the overall accelerator performance involves a trade-off between minimizing bunch energy spread and maximizing the efficiency [9,10]. As the growth of energy spread is due to the variation of the accelerating field along the length of the bunch, injecting short bunches is the obvious way to reduce the energy spread. However, optimum efficiency requires the extraction of a substantial fraction of the wake energy. In the limit of 100% efficiency, the bunch wake exactly cancels that of the laser pulse. In this case, the accelerating field falls to zero at the rear of the bunch, which again leads to a very large energy spread. Clearly efficient energy extraction is incompatible with a uniform accelerating field along the bunch, no matter how short it can be made.

A general strategy for minimizing the energy spread is the following [14]: as the bunch slips towards the zero of the accelerating field, the typical distribution of the laser wakefield is such that the rear part of the bunch experiences a higher accelerating gradient than the front. Note however, that the bunch wakefield usually causes a deceleration of the rear part of the bunch and therefore flattens the accelerating field profile, effectively suppressing the growth of energy spread. The challenge is to find a bunch length and a bunch charge consistent with a flat accelerating profile to minimize the growth of energy spread.

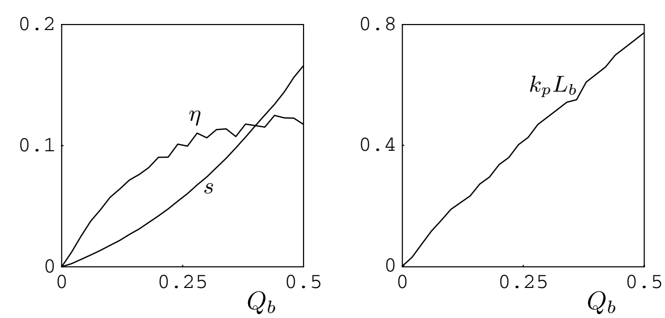


FIG. 4. Trade-off between energy spread and efficiency. Bunch length $L_b$, efficiency $\eta$ and relative energy spread $s$ are shown as functions of bunch charge $Q_b$. For each value of bunch charge, injection phase and bunch length have been varied to optimize the result (see text). Other parameters $a_0^2 = 1$, $k_p L_l = 1$, $k_0/k_p = 50$.

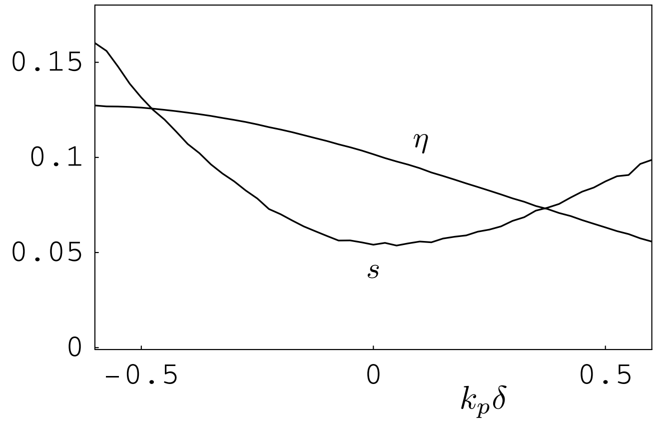


FIG. 5. Sensitivity to timing between laser pulse and electron bunch. Efficiency $\eta$ and relative energy spread $s$ as functions of the difference $\delta$ between the actual injection phase $\zeta_b$ and its optimized value. Other parameters $Q_b = 0.24$, $k_pL_b = 0.4$, $t/t_d = 0.4$, $a_0^2 = 1$, $k_pL_l = 1$, $k_0/k_p = 50$.

In Fig. 4 we plot the optimized bunch length $L_b$, the relative energy spread $s$ (i.e., relative to the average energy gain) and the efficiency $\eta$ for $a_0^2 = 1$, $k_pL_l = 1$, $k_0/k_p = 50$, all as functions of the dimensionless bunch charge $Q_b = k_p \int (n_b/n_p) d\zeta = \sqrt{\pi}(n_{b0}/n_p)k_pL_b$. We have used the following optimization strategy: to maximize $\eta$, the energy spread $s$ is evaluated at the time of maximum energy gain in each simulation. Subsequently, the value of $s$ is minimized by variation of both $\zeta_b$ and $L_b$ for each $Q_b$. The limit $Q_b \rightarrow 0$ corresponds to a single test electron, as the bunch length, efficiency and energy spread all approach 0 in this limit. As stated before, beam loading limits the amount of charge that can be accelerated, which explains why the efficiency tends to saturate with increasing bunch charge. Figure 4 also reveals the inherent trade-off mentioned above: increase of efficiency with increasing charge comes at the expense of an increase in energy spread.

Furthermore, the result is very sensitive to timing errors between the laser pulse and the electron bunch. A deviation of less than 10% of the plasma period can be sufficient to double the energy spread or significantly reduce the efficiency. This is demonstrated in Fig. 5, which shows the energy spread and efficiency as functions of the difference $\delta$ between the actual injection phase $\zeta_b$ and its optimal value for $Q_b = 0.24$, $k_pL_b = 0.4$, $t/t_d = 0.4$.

To conclude, we have given a fully self-consistent description of the coupled evolution of the laser pulse, the wakefield and the electron bunch in a laser-wakefield accelerator in one-dimensional geometry. We have studied the conflicting requirements for achieving efficient energy transfer from laser pulse to electron bunch, combined with a low energy spread. Our results show that a short pulse gives higher efficiency than a long pulse with the same energy. We have shown that the growth of energy spread can be suppressed by optimizing the relation between bunch length and bunch charge such that the variation of the accelerating field along the bunch is minimized. With such an optimal relation between bunch length and bunch charge, a possible compromise between minimizing energy spread and maximizing efficiency is found. Correct timing between the laser pulse and the electron bunch is found to be important, as energy spread and efficiency are very sensitive to errors in the injection phase.

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