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# Spontaneous coherent cyclotron emission from a short laser-kicked electron buncn

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## Abstract

Ponderomotive kicking of an electron bunch by the field of a laser pulse is proposed as a method for generating coherent cyclotron emission. It is shown that the imparted gyro-rotation can provide selective RF generation in an oversized microwave system, which can be rapidly tuned over a broad frequency range. A possible realization of a moderately relativistic source of short sub-millimeter wavelength pulses is studied. © 2004 Elsevier B.V. All rights reserved.

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## 1. Introduction

Photoinjectors are commonly used as a front end in modern accelerators to produce lowemittance high-charge sub-picosecond bunches of relativistic electrons. The availability of ultra-short e-bunches opens up the attractive possibility for spontaneous coherent radiation in the sub-millimeter and the far-infrared wavelength range [1,2]. Coherent radiation is emitted when all electrons of the bunch have approximately the same phase with respect to the radiated wave. For devices that depend on longitudinal electron bunching, such as

the ubitron free-electron maser, bunching on a wavelength scale or an e-bunch length shorter than half a wavelength is necessary to achieve coherent emission. The latter requirement is difficult to realize for radiation frequencies higher than a few 100 GHz because of the challenge of producing and transporting ultra-short high-charge ebunches while conserving their duration in the presence of space-charge forces. A completely different scenario exists for cyclotron masers, where electron bunching has a mixed transverselongitudinal character. This relaxes the need for ultra-short e-bunches because electron gyro-phases are not correlated longitudinally [3]. The ideal ebunch represents a rotating helix with the rotation frequency and wavenumber matched to the corresponding parameters of the radiation wave.

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In this work, we propose to use a laser pulse as a kicker to impart transverse (rotatory) motion to the electrons of a short bunch. This provides the proper correlation of phases of electron gyrorotation and, therefore, the coherent character of the emitted cyclotron radiation.

#### 2. Laser-kicked electron bunch

We consider an e-bunch moving rectilinearly in a uniform magnetic field with a velocity  $v_0$ . A short laser pulse propagates parallel to the e-bunch at the speed of light, c (Fig. 1). As an example, we consider a laser pulse with linear polarization,  $E_x = B_y = A \sin \theta_L$  where  $\theta_L = \omega_L t - k_L z$  and  $k_L = \omega_L/c$ . A displacement between the e-bunch and the axis of the transverse distribution of laser field, A(x, y), results in a ponderomotive force,  $F_L$ , which drives the electrons in the direction of diminishing laser field (Fig. 1a). Thus, the laser pulse initiates electron cyclotron rotation and transforms the rectilinear e-bunch into a helically rotating e-bunch (Fig. 1).

Electrons of the e-bunch following laser kicking have a longitudinal velocity,  $v_z < v_0$ , and a rotatory



Fig. 1. Pondermotive force provided by the laser pulse (a), and e-bunch before (b) and following (c) laser kicking.

velocity,  $v_+ = v_x + iv_y = v_\perp \exp i\theta_e$ , where  $\theta_e = \Omega(t - t_{kick}), \Omega = eB_0/\gamma mc$  is the cyclotron frequency and  $t_{kick}$  is the time representing the start of kicking. If at the time t=0 the electrons are distributed over an interval  $0 < z_0 < L_{e0}$  and the laser pulse is situated at the point z = 0, then  $t_{kick} = z_0/(c - v_0)$  and the current electron coordinate is expressed as  $z = z_0 + v_0 t_{kick} + v_z(t - t_{kick})$ . Thus, the electron gyro-phase can be expressed in the following form:

$$\theta_e = \frac{\Omega}{1 - \beta_z} t - \frac{\Omega/c}{1 - \beta_z} z \tag{1}$$

with  $\beta_z = v_z/c$ . Thus, laser kicking provides a spatio-temporal modulation of the e-bunch with the frequency and the wavenumber, which are close to parameters of a cyclotron-resonant wave

$$\omega = \frac{\Omega}{1 - \beta_z / \beta_\phi}, \quad h = \frac{\Omega / (\beta_\phi c)}{1 - \beta_z / \beta_\phi} \tag{2}$$

where  $\beta_{\phi} = \omega/hc$ . If the wave phase velocity is equal to the speed of light,  $\beta_{\phi} = 1$ , then exact synchronization appears: all electrons have the same gyro-phases with respect to the wave. This enables spontaneous coherent cyclotron radiation over the whole e-bunch. In a waveguide  $\beta_{\phi} > 1$ , and the synchronization is only approximate. The condition of phase synchronization  $\Delta\theta < \pi/2$  (here  $\theta = \omega t - hz - \theta_e$  is the electron phase with respect to the wave and  $\Delta\theta$  is the phase difference between the e-bunch edges) results in limitation of the ebunch length following kicking:

$$\frac{L_{\rm e}}{\lambda} < \frac{1 - \beta_z}{4(1 - \beta_{\phi}^{-1})}.$$
(3)

This condition can be used for controlling the mode of coherent cyclotron emission. If  $L_e$  is sufficiently long, then the e-bunch radiates mainly in the lowest possible transverse mode, which has the smallest  $\beta_{\phi}$  and, therefore, the highest frequency corresponding to the cyclotron resonance.

If laser frequency is far from cyclotron resonance, then the electron motion consists of fast oscillations in the laser field and the averaged slow motion. The latter is described by the equations

$$\frac{\mathrm{d}\,\bar{v}_x}{\mathrm{d}t} = F_{\mathrm{L}}(t) - \bar{v}_y \Omega, \ \frac{\mathrm{d}\,\bar{v}_y}{\mathrm{d}t} = \bar{v}_x \Omega, \tag{4}$$

where

$$F_{\rm L} = \frac{-\alpha^2}{4\omega_{\rm L}^2(1-\Omega^2/\omega_{\rm L}^{\prime 2})\,\partial x}$$

and  $\alpha = e/m\gamma$ . For a Gaussian shape  $F_{\rm L}(t) = F_0 \exp(-4t^2/t_{\rm L}^2)$ , the solution of Eqs. (4) has the form

$$\bar{v}_{+} = -\exp[i\Omega(t - t_{kick})]\frac{F_{0}}{\Omega}S(\omega)$$
(5)

where  $\psi = \Omega t'_L$ ,  $S(\psi) = \psi \exp(-\psi^2/16)\sqrt{\pi}/2$ , and  $t'_L = t_L/(1 - \beta_z)$ . The optimal phase,  $\psi \sim \pi$ , corresponds to the laser pulse duration

$$t_{\rm L} \approx \pi (1 - \beta_z) / \Omega \approx \pi / \omega \tag{6}$$

In the case of a laser pulse with a Gaussian transverse distribution,  $E^2(r) \propto \exp(-r^2/r_{\rm L}^2)$ , Eq. (5) leads to the following estimate

$$K = \gamma v_{\perp} / c \approx 2.5 \sqrt[4]{P_{\rm L}({\rm TW})} \sqrt{\lambda_{\rm L} / \lambda_{\rm c}}$$
(7)

where  $P_{\rm L}$  is the laser power,  $\lambda_{\rm c} = 2\pi c^2/\Omega\gamma$  is the wavelength corresponding to the non-relativistic cyclotron frequency, and  $\lambda_{\rm L}$  is the laser wavelength. Here we have assumed that the transverse electron oscillation is confined within the region of effective force  $F_{\rm L}$ :  $2r_{\rm c} \approx r_{\rm L}$ , where  $r_{\rm c}$  is the electron Larmor radius. It is important that the imparted gyro-velocity weakly depends on the resonant frequency  $\omega$ , which allows broadband fast frequency tuning.

#### 3. Simulation for the TOPS e-bunch

We study cyclotron emission from a laserkicked e-bunch in a circular waveguide, and assume that after kicking the particles rotate around the waveguide axis. In this case, only transverse waveguide modes that have the azimuthal index equal to unity can be excited at the fundamental cyclotron resonance [4]. Assuming a small electron gyro-radius compared with the waveguide radius we obtain the motion equations averaged over fast gyro-rotation [5], which are valid after

kicking 
$$(t > t_{kick})$$
:  
 $\frac{d\gamma}{d\tau} = -\beta_{\perp} e \sum_{m} Re \ a_m \exp(i\theta_m) - \beta \hat{\mathbf{E}}_{ch} ,$   
 $\frac{dp_z}{d\tau} = -\sum_{m} Re \ \chi_m a_m \exp(i\theta_m) - \mathbf{z_0} \hat{\mathbf{E}}_{ch}$ 

Here  $\tau = ct$ ,  $\boldsymbol{\beta} = \mathbf{v}/c$ ,  $p_z = \gamma \beta_z$ ,  $\theta_m = \omega_m t - h_m z - \theta_e$ ,  $\omega_m$  and  $h_m$  correspond to the resonance condition (2),  $\hat{\mathbf{E}}_{ch}$  is the normalized space-charge field,  $\chi_m = \beta_{\phi_m}^{\pm 1}$  for TE (–) and TM (+) modes,  $a_m$  is the *m*-th transverse mode amplitude described by the following equations

$$(1 - \beta_{z0}\beta_{\phi_m})\frac{\partial a_m}{\partial \zeta} + \frac{\partial a_m}{\partial \tau} = G_m \left\langle \frac{\beta_{\perp}}{\beta_z} \exp(-i\theta_m) \right\rangle_{\varphi}$$

Here  $\zeta = z - \beta_{z0}\tau$ ,  $G_m = Ie/mc^3 \cdot k_{\perp}^2 m/S_m \chi_m$ , *I* is the electron current,  $k_{\perp}$  is the transverse wavenumber, and *S* is the wave norm. We assume that at the moment  $t = t_{kick}$  initial electron gyrophases,  $\varphi$ , are distributed homogeneously over the interval  $0 < \varphi < \varphi_0$ . In simulation a quite big phase size,  $\varphi_0 = 0.2\pi$ , is taken.

We study possibility for realizing a moderately relativistic sub-millimeter wavelength frequencytunable source utilising the TOPS e-bunch source [6]. We consider a 2 nC 1 MeV e-bunch moving in a waveguide with guite a large diameter (5 mm), and study the excitation of the two lowest modes  $(TE_{1,1} \text{ and } TM_{1,1})$ . The magnetic field is chosen to correspond to the non-relativistic cyclotron frequency of 100 GHz ( $\lambda_c = 3 \text{ mm}$ ) so as to study a quite high ( $\sim \gamma^2$ ) Doppler frequency up-conversion. For these parameters and for the IR laser pulse with  $\lambda_{\rm L} = 1 \ \mu m$  and  $P_{\rm L} = 1 - 3 \ {\rm TW}$ , estimate (7) gives K = 0.14 - 0.22. Fig. 2 illustrate the shapes of the output RF pulses of the modes in the case of K = 0.17. In this case, the length of the operating region is about 20 cm. If the e-bunch length is short, then the both waves are excited. An increase of  $L_{\rm e}$  results in a fast decrease of the TMwave power and, simultaneously, in a significantly slower decrease of the TE-wave peak, so that at  $L_{\rm e} > 3$  mm practically single-mode emission occurs, which is consistent with the condition (3). Fig. 3 illustrates the possibility for frequency tuning by changing the value of the magnetic field for  $L_e = 3$  mm. The TE-mode pulse power depends



Fig. 2. Output power of the TE (a) and the TM (b) waves versus the time at various e-bunch lengths.



Fig. 3. Wavelength and output peak power of the TE wave, as well as output peak power of the TM wave versus the magnetic field (measured in wavelengths).

on the magnetic field very weakly. However, at short wavelengths, the TM-wave admixture becomes significant.

# 4. Conclusion

As a conclusion, we discuss the advantages of laser-kicked e-bunch cyclotron emission. First of all, in a relatively long and, therefore, low-density e-bunch, it provides a spatio-temporal modulation, which is almost ideal for coherent cyclotron emission. Second, good mode selectivity can be provided in a simple oversized microwave system. Actually, in addition to azimuthal selectivity imposed by an axis-encircling e-bunch, there is also a mechanism which provides the best condition for radiation in the lowest transverse mode. Third, since the laser kicking has a non-resonant character and, therefore, can be provided in a wide range of parameters, it is possible to provide fast frequency tuning by simply changing the resonant cyclotron frequency.

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