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# Generation of ultra-short quasi-unipolar electromagnetic pulses from quasi-planar electron bunches

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## Abstract

A method for the generation of quasi-unipolar pulses based on coherent synchrotron radiation from a quasi-planar electron bunch moving along a curved trajectory is proposed and theoretically studied. It is demonstrated that the experimental realization of this method at an existing installation (Terahertz to Optical Pulse Source) can result in generation of picosecond pulses with a peak power of up to 200 MW. © 2001 Elsevier Science B.V. All rights reserved.

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## 1. Introduction

Coherent spontaneous emission in free-electron lasers provides a method of generating short pulses of far-infrared radiation [1,2] However, to obtain ultra-broadband quasi-unipolar pulses a short undulator should be used to optimize the emission bandwidth. In the limit of half an undulator period a bandwidth of  $\delta\lambda/\lambda \sim 1$  should be possible. In this paper, we examine methods of generating intense quasi-unipolar pulses using a “pancake” shaped relativistic electron beam acting as a phased antenna array in a half-period undulator.

Photoinjectors are commonly used as a front end in modern accelerators to produce low emittance high charge sub-picosecond bunches of

relativistic electrons at very high powers, up to  $10^9$ – $10^{13}$  W. The availability of ultra-short electron bunches opens up the attractive possibility of producing quasi-unipolar electromagnetic pulses with durations comparable or even shorter than the duration of the electron pulse. It is clear from general principles that the pulse shape follows that of the electron current pulse. Here, we consider unipolar synchrotron radiation from a relativistic electron bunch moving along a finite curved trajectory, whose form determines the field of the radiated pulse. To ensure high directionality of the radiation, a quasi-planar bunch with transverse dimensions much larger than its longitudinal dimension is necessary to provide a “phased antenna array” pattern of the emitted radiation. A short length of the bunch provides a coherent summation of fields (rather than powers) emitted by particles (coherent radiation). According to the theory, a powerful unipolar electromagnetic pulse

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can be generated from a bunch emitting as it briefly executes an imposed transverse motion due to a magnetic field.

## 2. Basic principles

We begin with the well-known [3] example of radiation emitted by a single charged plane, which moves only along the  $x$ -direction for a finite interval of time with an imposed velocity  $v_x(t)$  (Fig. 1a). The plane radiates a pulse composed of  $E_x$  and  $B_y$  components of electric and magnetic fields, which are described by the Maxwell equations,

$$\frac{\partial E_x}{\partial z} = -\frac{1}{c} \frac{\partial B_y}{\partial t}, \quad \frac{\partial B_y}{\partial z} = -\frac{1}{c} \frac{\partial E_x}{\partial t} - \frac{4\pi}{c} j_x(t, z) \quad (1)$$

where  $j_x(t, z) = \sigma v_x(t) \delta(z)$ , and  $\sigma$  is the surface charge density. The plane radiates two identical pulses propagating in the  $\pm z$ -directions, with spatio-temporal profiles reproducing the velocity “profile”,  $v_x(t)$ :

$$E_x(t, z) = -\frac{2\pi\sigma}{c} \begin{cases} v_x(t - z/c), & z > 0, \\ v_x(t + z/c), & z < 0, \end{cases}$$

$$B_y(t, z) = \begin{cases} E_x(t, z) & z > 0, \\ -E_x(t, z) & z < 0. \end{cases}$$

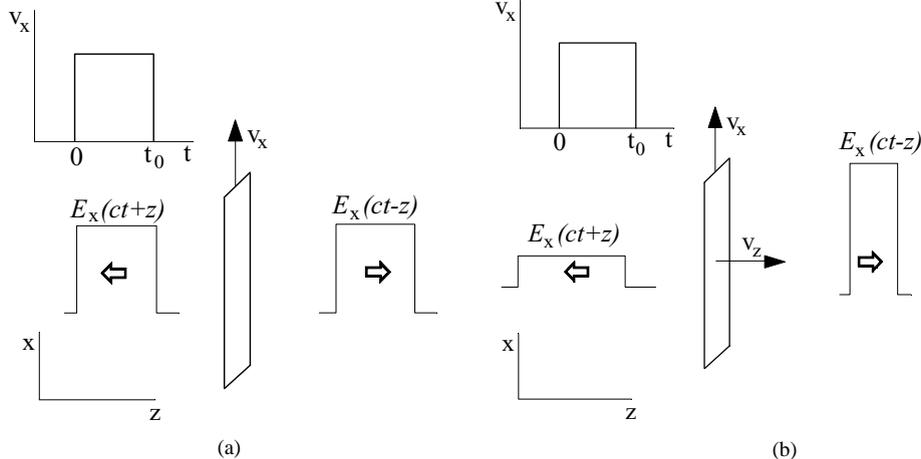


Fig. 1. Radiation of a moving charged plane: the plane does not move along  $z$ -coordinate (a) and moves with a velocity close to the speed of light (b).

Thus, monotonic motion of the charge plane in the  $x$ -direction (i.e. when the sign of  $v_x(t)$  does not change) results in radiation of a unipolar pulse.

An analogous result is produced when the plane moves not only in the  $x$ -direction, but also along the  $z$ -coordinate with a constant longitudinal velocity,  $v_z$  (Fig. 1b) [4]:

$$E_x(t, z) = -\frac{2\pi\sigma}{c} \begin{cases} \frac{v_x(\zeta_+)}{1 - \beta_z}, & z > v_z t, \\ \frac{v_x(\zeta_-)}{1 + \beta_z}, & z < v_z t, \end{cases}$$

$$B_y(t, z) = \begin{cases} E_x(t, z), & z > v_z t, \\ -E_x(t, z), & z < v_z t. \end{cases} \quad (3)$$

Here  $\beta_z = v_z/c$ , and  $\zeta_{\pm} = (t \mp z/c)/(1 \mp \beta_z)$ . At  $v_z \sim c$  the peak power of the forward pulse is significantly higher than that of the backward pulse. Due to the Doppler effect, the duration of the forward pulse is contracted, whereas the backward pulse is stretched.

Obviously, similar electromagnetic pulses can be generated from a quasi-planar moving electron bunch (Fig. 2), whose longitudinal ( $z$ -) dimension is much shorter than its transverse ( $x$ - and  $y$ -) dimensions. Such a bunch radiates mainly in the  $z$ -direction, as from a phased antenna array. A

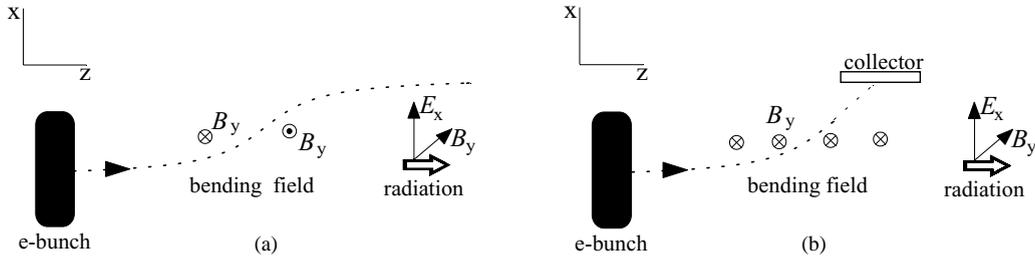


Fig. 2. Trajectories of a quasi-planar electron bunch moving through bending systems: two bending magnets (a), and a uniform magnetic field with a collector (b).

magnetostatic field bends the bunch trajectory so that during a finite interval of the time the bunch has a positive  $x$ -component of velocity. The magnetic field could be due to two bending magnets providing a translation of the bunch in the  $x$ -direction (Fig. 2a). Another possibility could be a single bending magnet followed by a collector (Fig. 2b); the collector is necessary to stop the motion of the electron bunch along the  $x$ -direction and, therefore, to control the duration and polarization of the radiated pulse. In both cases, the “pulse” of transverse motion of the bunch results in synchrotron radiation with linear  $(E_x, B_y)$  polarization. Since on the entire bunch trajectory the transverse velocity does not change its sign, the radiation has a unipolar character. The forward-pulse duration is estimated using the Doppler-converted time of the radiation,

$$t_p \approx L_b(1 - \bar{v}_z/c)/\bar{v}_z \quad (4)$$

where  $L_b$  is the length of the bending system, and  $\bar{v}_z$  is the characteristic transnational velocity of the bunch in the bending region. Thus, the use of an ultra-relativistic electron bunch enables the production of ultra-short electromagnetic pulses.

### 3. Simulations for the TOPS project

As a concrete example, we consider the ultra-short pulse electron source at the Terahertz to Optical Pulse Source (TOPS) developing at the Strathclyde University [5]. This facility will produce quasi-planar electron bunches with a total charge of the order of 1 nC and an energy

of 3–4 MeV. At an electron energy of 3.2 MeV, the Doppler conversion factor,  $\Gamma = 1/(1 - v_z/c) \approx 2\gamma^2$ , is of about 100. Here  $\gamma$  is the relativistic Lorentz-factor of the electrons. According to Eq. (4), the bending system length of few centimeters corresponds to radiation of a quasi-unipolar pulse with a duration  $t_p \sim 1$  ps. Such a pulse has a central frequency of  $f_c \approx 1/t_p \sim 1$  THz and a bandwidth of the order of  $f_c$ .

For the TOPS project, we simulate the radiation of a quasi-planar cylindrical electron bunch with a  $z$ -dimension of 0.3 mm and a transverse diameter of 1.4 mm, which initially moves rectilinearly along the  $z$ -coordinate. A finite-extent magnetostatic system provides a bending field,  $\mathbf{B}_b = \mathbf{y}B_b$ , driving the electrons along the  $x$ -coordinate. The electron motion is described by the following equations for the relativistic electron momentum,  $\mathbf{p} = m\gamma\mathbf{v}$ ,

$$\begin{aligned} \frac{dp_x}{dt} &= -eE_x + \frac{e}{c}v_z(B_y + B_b), \\ \frac{dp_z}{dt} &= -eE_c - \frac{e}{c}v_x(B_y + B_b). \end{aligned} \quad (5)$$

Here  $E_x$  and  $B_y$  are electric and magnetic components of the radiated field, and  $E_c$  is the  $z$ -component of the Coulomb field. The Coulomb repulsion of electrons in transverse directions is neglected because it is partially compensated by the magnetic attraction of parallel electron currents. We assume that the bending field is weak enough to result in a negligibly small change in longitudinal electron velocity. Then the temporal derivative in Eq. (5) is transformed into the form  $d/dt \approx \partial/\partial t + v_0\partial/\partial z$ , where  $v_0$  is the initial electron velocity. The variables  $t$  and  $z$  are then

substituted by new normalized variables,  $\zeta = (z - v_0 t)/L_b$  and  $\tau = ct/L_b$ . This leads to the following equations for the normalized  $x$ - and  $z$ -components of the electron momentum,

$$\begin{aligned} \frac{\partial(\gamma\beta_x)}{\partial\tau} &= -\hat{E}_x + \beta_z(\hat{B}_y + \hat{B}_b), \\ \frac{\partial(\gamma\beta_z)}{\partial\tau} &= -\hat{E}_z - \beta_x(\hat{B}_y + \hat{B}_b) \end{aligned} \quad (6)$$

and the following equations for the electron coordinates,

$$\frac{\partial\hat{x}}{\partial\tau} = \beta_x, \quad \frac{\partial\zeta}{\partial\tau} = \beta_z - \beta_0 \quad (7)$$

where  $\hat{x} = x/L_b$ ,  $\beta_{x,z} = v_{x,z}/c$  are the electron velocities normalized by the speed of light,  $\beta_0 = v_0/c$ , and  $(\hat{E}, \hat{B}) = (eL_b/mc^2)(E, B)$  are the normalized fields.

The Maxwell equations (1) are transformed into the following form:

$$\begin{aligned} \frac{\partial\hat{E}_x}{\partial\zeta} &= -\frac{\partial\hat{B}_y}{\partial\tau} + \beta_0\frac{\partial\hat{B}_y}{\partial\zeta}, & \frac{\partial\hat{B}_y}{\partial\zeta} &= -\frac{\partial\hat{E}_x}{\partial\tau} + \beta_0\frac{\partial\hat{E}_x}{\partial\zeta} \\ &- \frac{4\pi L_b^2}{I_A} j_x(\tau, \zeta) \end{aligned} \quad (8)$$

where  $I_A = mc^3/e$ . The boundary conditions for Eqs. (8) can be formulated as radiation conditions in vacuum,

$$\frac{E_x}{B_y} = \begin{cases} 1, & \zeta > \zeta_{\max} \\ -1, & \zeta < \zeta_{\min} \end{cases} \quad (9)$$

where  $\zeta_{\max}$  and  $\zeta_{\min}$  are the extrema of the electrons in the bunch

We first consider a magnetostatic system in the form of two bending magnets (Fig. 2a), which can be modeled by the following  $y$ -component of the magnetic field,

$$B_b = B_0 \cos(\pi z/L_b), \quad 0 \leq z \leq L_b. \quad (10)$$

The results of simulations of Eqs. (5)–(9) for the bending field Eq. (10) and the length of the bending system  $L_b = 1.2$  cm are shown in Fig. 3. For relatively small values of the bending field the forward radiated pulse is quasi-unipolar with duration of 1–2 ps. The power of the emitted radiation increases with the increase of the bending field, with a maximum of 200 MW achievable for a bending field magnitude of  $B_0 = 8$  kGs. This is

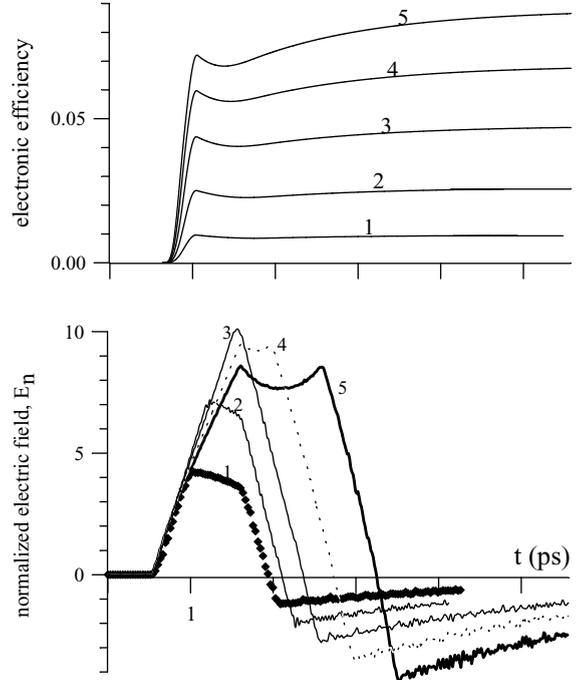


Fig. 3. Synchrotron radiation of an electron bunch during its motion through a two-magnet bending system. Electronic efficiency and radiated electric field versus time in the cases of magnitudes of the bending field,  $B_0$ , 3.3 kGs (1), 5.5 kGs (2), 8.0 kGs (3), 10.0 kGs (4), and 12.0 kGs (5). The normalized field,  $E_n$ , is connected with the radiated power by the formula  $P = 2 \text{ MW } E_n^2$ .

accompanied by an increase of the pulse duration, which can be explained by two effects. The first one is an increase of the transverse electron velocity in the bending region, and, therefore, a decrease of the Doppler conversion factor,  $\Gamma = (1 - \bar{v}_z/c)^{-1}$ . The second effect is a perturbation of the electron motion by the radiated and Coulomb fields (radiation reaction and Coulomb repulsion). For bending field over 8 kGs, these effects lead to a saturation of the radiation process: any further increase of  $B_0$  results in a decrease of the peak power accompanied by an increase in the duration of the radiated pulse.

At high bending fields the radiated pulse becomes bipolar. The electric field at the rear of the pulse reverses direction and results in a long “tail” of weak field having a “wrong” polarization. The origin of the negative “tail” is the transverse motion of electrons in the region after the bending system.

This phenomenon is caused by the collective effect of the interaction of different charge planes of the electron bunch, namely, the influence of the radiated field on the transverse motion of electrons. Actually, according to Eqs. (2) or (3), the radiated electric field counteracts the electron  $x$ -motion imposed by the bending field. Thus, the counterforce of the radiation acts to decrease the transverse velocity. Moreover, in the region after the bending system, the radiated electric field drives the electrons in the opposite direction, due to the retardation of the radiation. This results in a more complicated form of the radiated pulse: its duration increases and its rear experiences a reverse of the polarization. For weak bending fields,  $B_0$ , the collective effect is unimportant. In contrast, large values of  $B_0$  modify the pulse shape into a bipolar form.

Evidently, the bipolar nature of the pulse can be avoided by collecting the bunch inside the bending

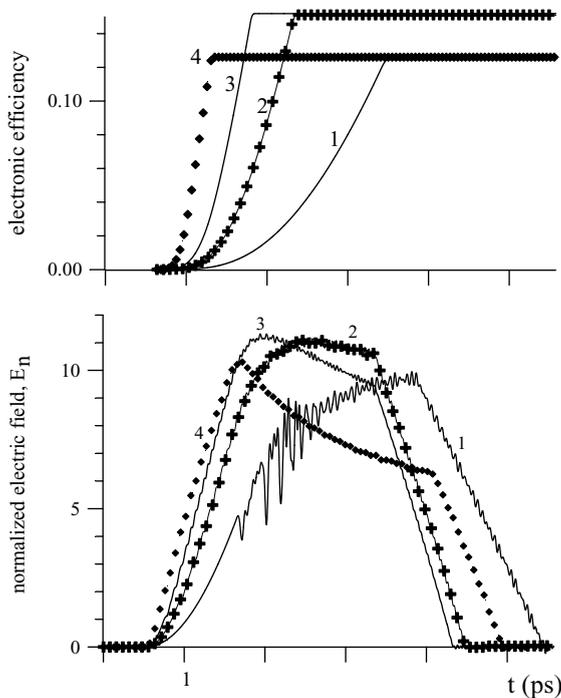


Fig. 4. Synchrotron radiation of TOPS electron bunch during its motion through a uniform bending field. Electronic efficiency and radiated electric field versus time in the cases of values of the bending field,  $B_b$ , 0.2 kGs (1), 0.5 kGs (2), 1.0 kGs (3), and 3.0 kGs (4). The normalized field,  $E_n$ , is connected with the radiated power by the formula  $P = 2 \text{ MW } E_n^2$ .

region (Fig. 2b). As an example, we consider the bunch motion in a uniform magnetic field,  $\mathbf{y}B_b$ . The electron motion is abruptly stopped by a collector placed at a distance of  $x = 0.5 \text{ cm}$  from the point of the bunch injection (in the simplest model we neglect the electron interaction with the collector, which could lead to undesired radiation). The output radiation represents an exactly unipolar pulse with power and duration depending on the value of the bending field,  $B_b$  (Fig. 4). Magnetic fields up to 1 kGs shorten the output electromagnetic pulse due to a decrease of the characteristic bending length. However, any further increase of the bending field results in an increase of the duration of the output pulse due to a decrease in the Doppler conversion factor.

#### 4. Conclusion

According to the theory, the proposed method enables the production of picosecond quasi-unipolar pulses with a high peak power. It should be noticed that in this work we have not considered propagation of the quasi-unipolar pulses. This is justified because the radiation is produced in a short interaction region (bending system) by a “pancake” shaped electron bunch. Thus, we have used a model of radiating charge planes in vacuum that neglects both diffraction of the pulse and dispersion of the medium. The influence of these effects can be minimized using, for example, a weakly dispersive strip-line waveguide.

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