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Electromagnetically induced transparency for two intense waves in plasma

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Abstract. The coupled propagation of two electromagnetic waves in plasma is studied in order to find the conditions for induced transparency. This means unattenuated propagation of the waves through plasma which is overdense, and thus opaque, for one (or both) of them. This is made possible by a modulation of the refractive properties of the plasma, due to a relativistic increase in the electron mass, or to a variation in electron density caused by longitudinal plasma oscillations driven by the ponderomotive potential associated with the beat of the waves. Starting from a relativistic fluid description, we make an *Ansatz* containing two transverse monochromatic electromagnetic plane waves, and longitudinal plasma oscillations at the sum and difference of their frequencies. For weakly relativistic intensities we derive coupled dispersion relations, which take into account the polarization of the waves and the nonlinearities with respect to both their amplitudes. This serves to explore the conditions for induced transparency and the modes of propagation.

1. Introduction

Induced transparency [1, 2], lasing without inversion [3] and related phenomena [4] stimulate the interest of researchers in their search for new nonlinear media suitable for amplifying or switching. This has particular appeal in the quest for coherent X-ray lasers, where mirrors are difficult to obtain.

Electromagnetically induced transparency (EIT) means the propagation of an electromagnetic wave through an otherwise opaque medium, made possible by the interaction with a second wave. In plasma, the coupling mechanism is a modulation of the plasma frequency, which determines the refractive properties of the plasma. This is due both to a relativistic increase in the electron mass and to a variation in electron density caused by longitudinal plasma oscillations driven by the ponderomotive potential associated with the beat of the waves.

A study of EIT by Harris [1] employed a three-wave model, incorporating two transverse electromagnetic waves, with frequencies $\omega_1 > \omega_p$, $\omega_2 < \omega_p$, where $\omega_p = (4\pi n_0 e^2/m)^{1/2}$ is the (unperturbed) plasma frequency (n_0 is the unperturbed electron density, $-e$ the electron charge and m their rest mass), and a longitudinal plasma wave at the difference frequency $\omega_- = \omega_1 - \omega_2$. It predicts transparency if the latter is slightly lower than ω_p.

Matsko and Rostovtsev [5] found that the conditions for EIT are affected by the excitation of the anti-Stokes wave at $\omega_1 + \omega_p$ as well.

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While these workers assumed that the amplitude of the wave at the lower frequency is small, an investigation of modulational instability of two coupled waves (which involves the same interaction mechanism) by McKinstrie and Bingham [6] allowed for finite amplitudes of both waves and also took the relativistic electron mass into account. They used an expansion of the wave vectors in terms of the amplitudes, which, however, breaks down at the transition to transparency.

The aim of the present paper is to explore the conditions of EIT for two waves of weakly relativistic amplitudes, which may be comparable. Moreover, we do not restrict the frequencies, so that either or both may be above or below the plasma frequency.

2. Coupled propagation in plasma

2.1. *Relativistic fluid equations*

We start from a description of the plasma as cold electron fluid. We take the relativistic corrections to the electron mass into account since they are of the same order of magnitude as the ponderomotive coupling [6]. In one spatial dimension (z) the relevant equatons are (see for example [7])

$$\left(\frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial t^2} - \frac{n}{\gamma}\right)\mathbf{a} = \mathbf{0}, \tag{1}$$

$$\frac{\partial E}{\partial t} = \frac{n}{\gamma}p, \quad \frac{\partial E}{\partial z} = 1 - n, \quad \frac{\partial p}{\partial t} = -E - \frac{\partial \gamma}{\partial z}, \tag{2}$$

where $\mathbf{a}(z,t)$ is the transverse vector potential scaled by mc^2/e (c is the vacuum speed of light), $E(z,t)$ is the longitudinal electric field scaled by $\omega_p mc^2/e$, $p(z,t)$ is the longitudinal momentum scaled by mc, $n(z,t)$ is the electron density scaled by the unperturbed density n_0 and $\gamma(z,t) = (1 + \mathbf{a}^2 + p^2)^{1/2}$ is the Lorentz factor. Time is scaled by $1/\omega_p$ and length by c/ω_p.

2.2. *Expansion in Powers of the Vector Potential*

In this section, we determine the coupling of two transverse waves, driven by applied fields of frequencies ω_1 and ω_2, through longitudinal oscillations to the lowest (second) order in an expansion in powers of the vector potential. For the latter, we try an *Ansatz* of the form

$$\mathbf{a} = \Re[\mathbf{a}_1 \exp(\mathrm{i}\theta_1) + \mathbf{a}_2 \exp(\mathrm{i}\theta_2)], \tag{3}$$

with complex amplitude vectors $\mathbf{a}_{1,2}$ and phases $\theta_{1,2} = k_{1,2}z - \omega_{1,2}t$. (Corresponding to the scalings of time and length, the frequencies $\omega_{1,2}$ are scaled by ω_p and the wavenumbers $k_{1,2}$ by ω_p/c.)

From equations (2) we find that the ponderomotive force $-\partial\gamma/\partial z$ drives longitudinal plasma waves, characterized by E, p and $n-1$. To the lowest order these quantities are proportional to the (scaled) intensity of the transverse waves

$$I = \mathbf{a}^2 = I_0^{(1)} + I_0^{(2)} + \Re(I_+ + I_- + I_{11} + I_{22}), \tag{4}$$

where the individual terms are given by

$$\left.\begin{aligned} I_0^{(1,2)} &= \frac{|\mathbf{a}_{1,2}|^2}{2}, & I_{11,22} &= \frac{\mathbf{a}_{1,2}^2 \exp(2\mathrm{i}\theta_{1,2})}{2}, \\ I_+ &= \mathbf{a}_1 \cdot \mathbf{a}_2 \exp(\mathrm{i}\theta_+), & I_- &= \mathbf{a}_1 \cdot \mathbf{a}_2^* \exp(\mathrm{i}\theta_-), \end{aligned}\right\} \tag{5}$$

with sum and difference phases $\theta_\pm = \theta_1 \pm \theta_2$.

The Lorentz factor, to this order, is $\gamma \approx (1+I)^{1/2} \approx 1 + I/2$, so that the density modulations are determined by

$$\left(\frac{\partial^2}{\partial t^2} + 1\right)(n-1) \approx \frac{1}{2}\frac{\partial^2 I}{\partial z^2}. \tag{6}$$

Accordingly, they may be decomposed as

$$n - 1 = \Re(n_+ + n_- + n_{11} + n_{22}), \tag{7}$$

with

$$n_\pm = \frac{k_\pm^2 I_\pm}{D_\pm}, \quad n_{11,22} = \frac{4k_{1,2}^2 I_{11,22}}{D_{11,22}}, \tag{8}$$

and $k_\pm = k_1 \pm k_2$, $D_\pm = \omega_\pm^2 - 1$, $D_{11,22} = 4\omega_{1,2}^2 - 1$, $\omega_\pm = \omega_1 \pm \omega_2$.

2.3. *Dispersion relation*

We substitute n and γ into equation (1) for the vector potential and retain only terms oscillating with the frequencies ω_1, ω_2 of the applied fields. (This assumes that the frequencies $3\omega_1$, $3\omega_2$, $|2\omega_1 + \omega_2|$ and $|\omega_1 + 2\omega_2|$ of the discarded terms differ from ω_1 and ω_2.) We thus arrive at coupled dispersion relations for the wavenumbers k_1 and k_2:

$$\left[\left(D_1 - k_1^2 + \frac{I_0^{(1)} + I_0^{(2)}}{2}\right)\mathbf{1} + \left(1 - \frac{k_+^2}{D_+}\right)\frac{\mathbf{a}_2^*\mathbf{a}_2}{4} + \left(1 - \frac{k_-^2}{D_-}\right)\frac{\mathbf{a}_2\mathbf{a}_2^*}{4} + \left(1 - \frac{4k_1^2}{D_{11}}\right)\frac{\mathbf{a}_1^*\mathbf{a}_1}{8}\right] \cdot \mathbf{a}_1 = 0, \tag{9}$$

$$\left[\left(D_2 - k_2^2 + \frac{I_0^{(1)} + I_0^{(2)}}{2}\right)\mathbf{1} + \left(1 - \frac{k_+^2}{D_+}\right)\frac{\mathbf{a}_1^*\mathbf{a}_1}{4} + \left(1 - \frac{k_-^2}{D_-}\right)\frac{\mathbf{a}_1\mathbf{a}_1^*}{4} + \left(1 - \frac{4k_2^2}{D_{22}}\right)\frac{\mathbf{a}_2^*\mathbf{a}_2}{8}\right] \cdot \mathbf{a}_2 = 0, \tag{10}$$

where $\mathbf{1}$ denotes a unit tensor and $D_{1,2} = \omega_{1,2}^2 - 1$.

The first term in each equation corresponds to a reduction of the plasma frequency by a factor $1 - (I_0^{(1)} + I_0^{(2)})/2$ due to the time-averaged relativistic increase in the electron mass. It is independent of the polarizations of the waves, in contrast to the subsequent terms with the dyads of amplitude vectors.

2.4. *Polarizations*

We split the amplitude vectors into scalar amplitudes and unit vectors, $\mathbf{a}_{1,2} = a_{1,2}\mathbf{e}_{1,2}$ and find that the *Ansatz* (3) works for four different combinations of polarizations:

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LP: linear polarizations, parallel: $\mathbf{e}_1 = \mathbf{e}_2 = \mathbf{e}_x$;
LO: linear polarizations, orthogonal: $\mathbf{e}_1 = \mathbf{e}_x, \mathbf{e}_2 = \mathbf{e}_y$;
CS: circular polarizations, same senses of rotation: $\mathbf{e}_1 = \mathbf{e}_2 = \mathbf{e}_+ = (\mathbf{e}_x + i\mathbf{e}_y)/2^{1/2}$;
CO: circular polarizations, opposite senses of rotation: $\mathbf{e}_1 = \mathbf{e}_+, \mathbf{e}_2 = \mathbf{e}_- = \mathbf{e}_+^*$.

In each case, the dispersion relation takes the form of two coupled bilinear equations in the wavenumbers k_1 and k_2:

$$A_{11}k_1^2 + 2A_{12}k_1k_2 + A_{22}k_2^2 = A, \quad B_{11}k_1^2 + 2B_{12}k_1k_2 + B_{22}k_2^2 = B, \tag{11}$$

with coefficients, depending on the polarizations:

LP:

$$A_{11} = (D_{11} + I_0^{(1)})D_+D_- + A_{22}, \quad A_{12} = \frac{I_0^{(2)}D_{11}(D_- - D_+)}{2},$$
$$A_{22} = \frac{I_0^{(2)}D_{11}(D_+ + D_-)}{2}, \quad A = D_{11}D_+D_-\left(D_1 + \frac{3I_0^{(1)}}{4} + \frac{3I_0^{(2)}}{2}\right),$$
$$B_{11} = \frac{I_0^{(1)}D_{22}(D_+ + D_-)}{2}, \qquad B_{12} = \frac{I_0^{(1)}D_{22}(D_- - D_+)}{2},$$
$$B_{22} = (D_{22} + I_0^{(2)})D_+D_- + B_{11}, \quad B = D_{22}D_+D_-\left(D_2 + \frac{3I_0^{(1)}}{2} + \frac{3I_0^{(2)}}{4}\right);$$

LO:

$$A_{11} = D_{11} + I_0^{(1)}, \quad A_{12} = A_{22} = 0, \quad A = D_{11}\left(D_1 + \frac{3I_0^{(1)}}{4} + \frac{I_0^{(2)}}{2}\right),$$
$$B_{11} = B_{12} = 0, \quad B_{22} = D_{22} + I_0^{(2)}, \quad B = D_{22}\left(D_2 + \frac{I_0^{(1)}}{2} + \frac{3I_0^{(2)}}{4}\right);$$

CS and CO:

$$A_{11} = D_\mp + \frac{I_0^{(2)}}{2}, \quad A_{12} = \mp A_{22} = \mp\frac{I_0^{(2)}}{2}, \quad A = D_\mp\left(D_1 + \frac{I_0^{(1)}}{2} + I_0^{(2)}\right),$$
$$B_{11} = \mp B_{12} = \frac{I_0^{(1)}}{2}, \quad B_{22} = D_\mp + \frac{I_0^{(1)}}{2}, \quad B = D_\mp\left(D_2 + I_0^{(1)} + \frac{I_0^{(2)}}{2}\right).$$

In the last set of coefficients, the upper sign is for CS, and the lower sign for CO.

Mathematically, equations (11) represent conic sections in the k_1–k_2 plane, centred at the origin, and can be solved analytically. It should be noted that for circular polarizations the longitudinal waves providing the coupling are excited only at the difference frequency ω_- for CS (as assumed in [1], although for linear polarizations) and at the sum frequency ω_+ for CO. For LO the equations are decoupled.

3. Results

3.1. *Conditions for transparency*

The plasma is transparent for both waves if, for real ω_1 and ω_2, the wavenumbers k_1 and k_2 are both real. We have checked this condition for the solution of the dispersion relations (11) for different amplitudes a_1 and a_2 and plotted the areas where it is satisfied for at least one of the two sets of solutions in the frequency ranges $\omega_1 = 0$–2.5 and $\omega_2 = 0$–1.1. Figure 1 shows the results for parallel linear polarizations and figure 2 the results for circular polarizations.

In figure 1 (*a*), a_2 is set to zero; this corresponds to the treatment by Harris [1], and Matsko and Rostovtsev [5], except that we take the second harmonic $2\omega_1$ into account. We notice a region of transparency and, for $\omega_{1,2} \geqslant 1$, a complementary region of induced opacity, each bounded by the resonance of the difference

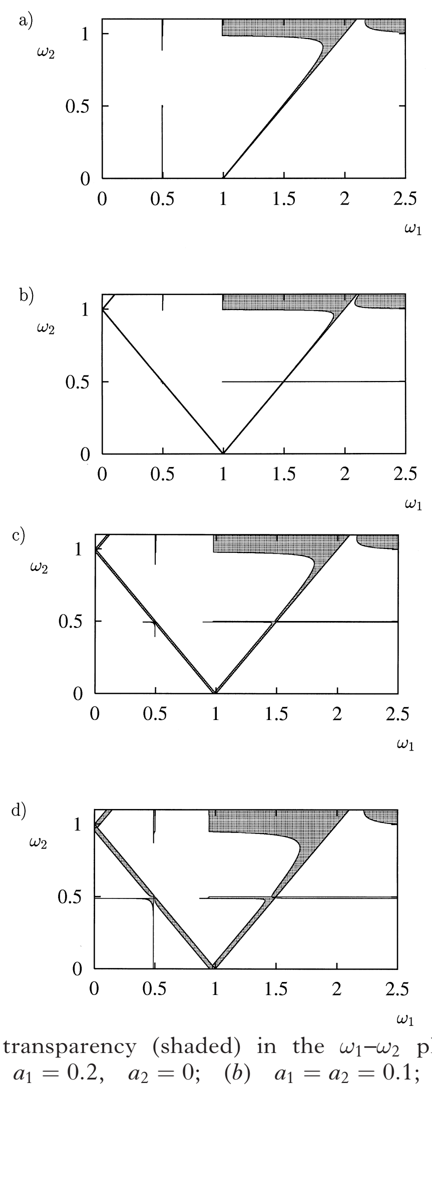

Figure 1. Regions of transparency (shaded) in the ω_1–ω_2 plane, for parallel linear polarizations: (*a*) $a_1 = 0.2$, $a_2 = 0$; (*b*) $a_1 = a_2 = 0.1$; (*c*) $a_1 = a_2 = 0.2$; (*d*) $a_1 = a_2 = 0.3$.

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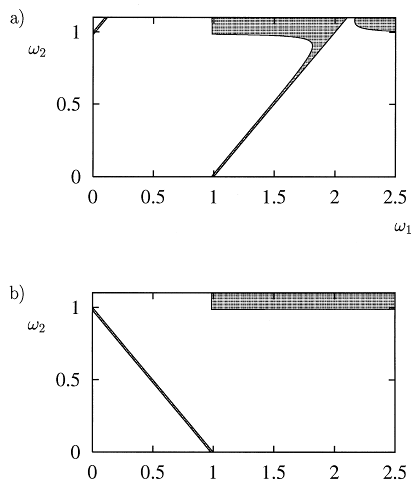

Figure 2. Regions of transparency (shaded) in the ω_1–ω_2 plane, for circular polarizations, $a_1 = a_2 = 0.2$: (a) co-rotating; (b) counter-rotating.

frequency with the plasma frequency, $\omega_- = 1$, as found by these workers. Additionally, we find transparency in a narrow region bounded by the second harmonic resonance, $2\omega_1 \leqslant 1$. Plots for equal amplitudes $a_1 = a_2 = 0.1$–0.3 (figures 1 (*b*)–(*d*)) show additional regions of transparency near the resonances of the sum frequency $\omega_+ = 1$, and also of the second harmonics, $2\omega_1 = 1$ and $2\omega_2 = 1$. All these regions grow in width as the amplitudes increase.

In figure 2 we see that for the cases of co-rotating and counter-rotating circular polarizations, transparency is induced only near the resonances of the difference and sum frequencies respectively.

3.2. *Dispersion curves*

Figure 3 shows examples of dispersion curves for the wavenumbers k_1 and k_2 versus ω_2 for fixed ω_1 in the case of parallel linear polarizations. Figures 3 (*a*)–(*c*) are cuts through the transparency regions near $\omega_- = 1$ and $2\omega_2 = 1$ for $\omega_1 = 1.45$ for different amplitude ratios. Also indicated in these plots is the wavenumber corresponding to the higher frequency: $k_1^{(0)} = (\omega_1^2 - 1)^{1/2}$. Figure 3 (*d*) is a cut through the regions near $\omega_+ = 1$ and $2\omega_2 = 1$ for $\omega_1 = 0.45$ and equal amplitudes $a_1 = a_2 = 0.3$. We note the existence of two modes with different wavenumbers for certain frequency ranges and the possibility of transparency for counter-propagating waves, indicated by opposite signs of k_1 and k_2.

4. Conclusions

We have studied the coupled propagation of two transverse electromagnetic plane waves through a weakly relativistic cold plasma, taking into account longitudinal plasma waves up to second order in the amplitudes of the transverse fields. These waves are excited at the sum and difference of the frequencies of the applied fields and, at their second harmonics, lead to transparency if either of these frequencies lies in a narrow band below the plasma frequency. These bands

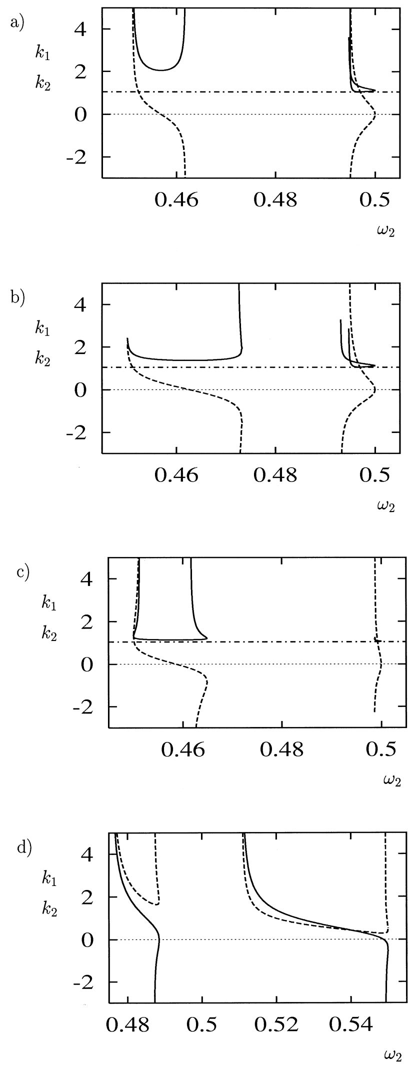

Figure 3. Dispersion curves of $k_1(\omega_2)$ (——), $k_2(\omega_2)$ (– – –) for fixed ω_1, parallel linear polarizations: (*a*) $\omega_1 = 1.45$, $a_1 = 0.1$ and $a_2 = 0.2$; (*b*) $\omega_1 = 1.45$ and $a_1 = a_2 = 0.2$; (*c*) $\omega_1 = 0.45$, $a_1 = 0.2$ and $a_2 = 0.1$; (*d*) $\omega_1 = 0.45$ and $a_1 = a_2 = 0.3$. In (*a*)–(*c*), $k_1^{(0)}$ (– · –) is also shown.

broaden as the amplitudes increase. For the difference frequency this has been described earlier by Harris [1] and Matsko and Rostovtsev [5]. By contrast, transparency induced through plasma oscillations at the sum frequency is of particular interest, since in this case neither transverse wave can propagate in the plasma on its own.

An important issue that remains to be addressed is the evolution of the field amplitudes when they are switched on; how do the waves penetrate from the surface into the plasma? Will the interaction of the evanescent waves near the surface gradually establish the longitudinal oscillations necessary for transparency? From this point of view, the theoretical possibility of transparency mutually induced by counter-propagating waves below the plasma frequency is very unlikely to be realized.

Since there is no restriction on the frequencies in the present formalism, it can equally be used to study other phenomena of nonlinear collective interaction in plasma, such as Raman scattering or modulational instability.

We hope to be able to complement the theory with experiments at the Strathclyde Electron and Terahertz to Optical Pulse Source (TOPS) [8] in the near future. In one set-up, we plan to use the 4 TW beam at TOPS (wavelength $\lambda_1 = 800\,\mathrm{nm}$) to reach scaled amplitudes a_1 of up to unity to induce transparency for a tuneable second beam with amplitude $a_2 \leqslant 0.1$, using the resonance of the difference frequency. This requires a plasma density of the order of $n_0 \approx 10^{21}\,\mathrm{cm}^{-3}$. A different set-up would use pulses in the terahertz range, for example from a free-electron laser and plasma of a much lower density $n_0 \approx 10^{15}\,\mathrm{cm}^{-3}$.

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